Fundamentals of Microelectronics II

فصل ۱۱- پاسخ فرکانسی

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Chapter 11 Frequency Response

- 11.1 Fundamental Concepts
- 11.2 High-Frequency Models of Transistors
- 11.3 Analysis Procedure
- 11.4 Frequency Response of CE and CS Stages
- 11.5 Frequency Response of CB and CG Stages
- 11.6 Frequency Response of Followers
- 11.7 Frequency Response of Cascode Stage
- 11.8 Frequency Response of Differential Pairs
- 11.9 Additional Examples

Chapter Outline

Fundamental Concepts

- Bode's Rules
- Association of Poles with Nodes
- Miller's Theorem

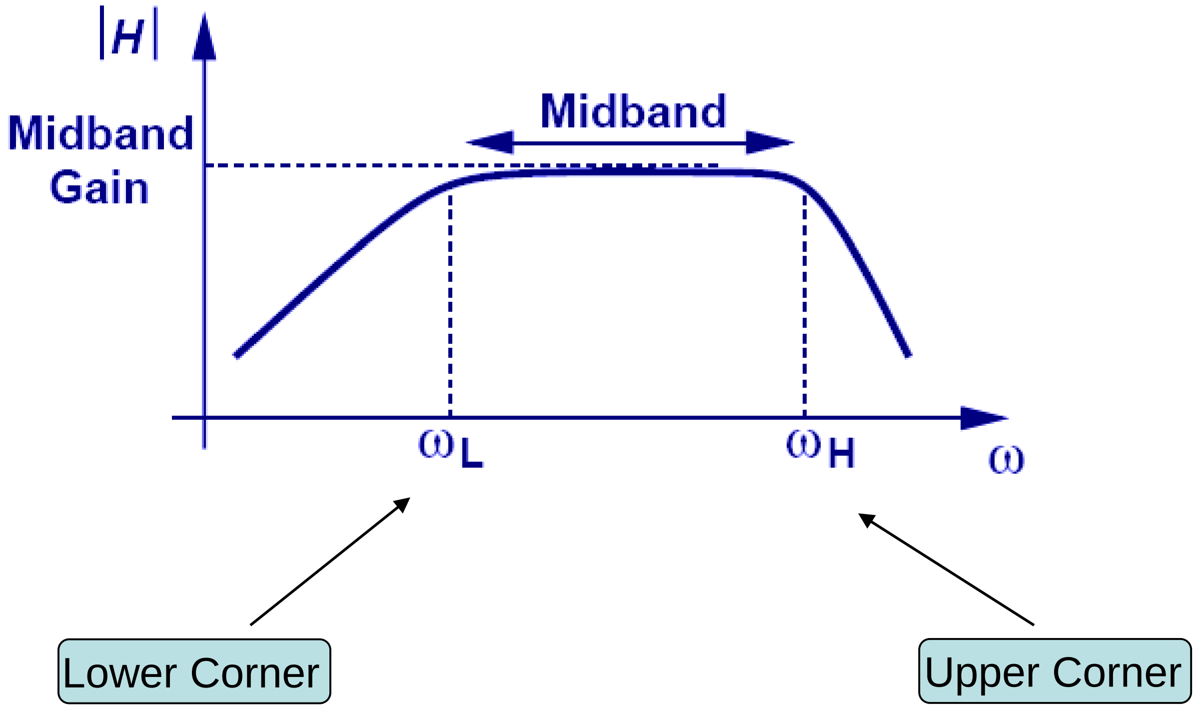
High-Frequency Models of Transistors

- Bipolar Model
- MOS Model
- Transit Frequency

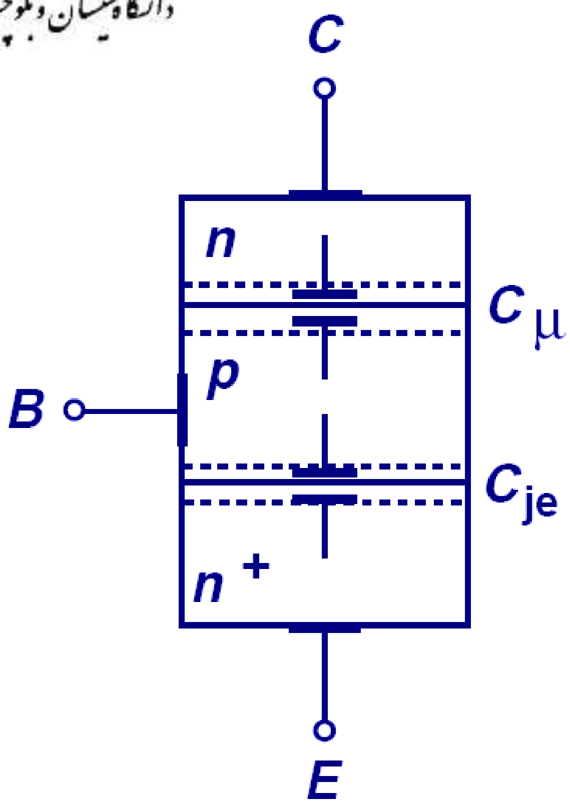
Frequency Response of Circuits

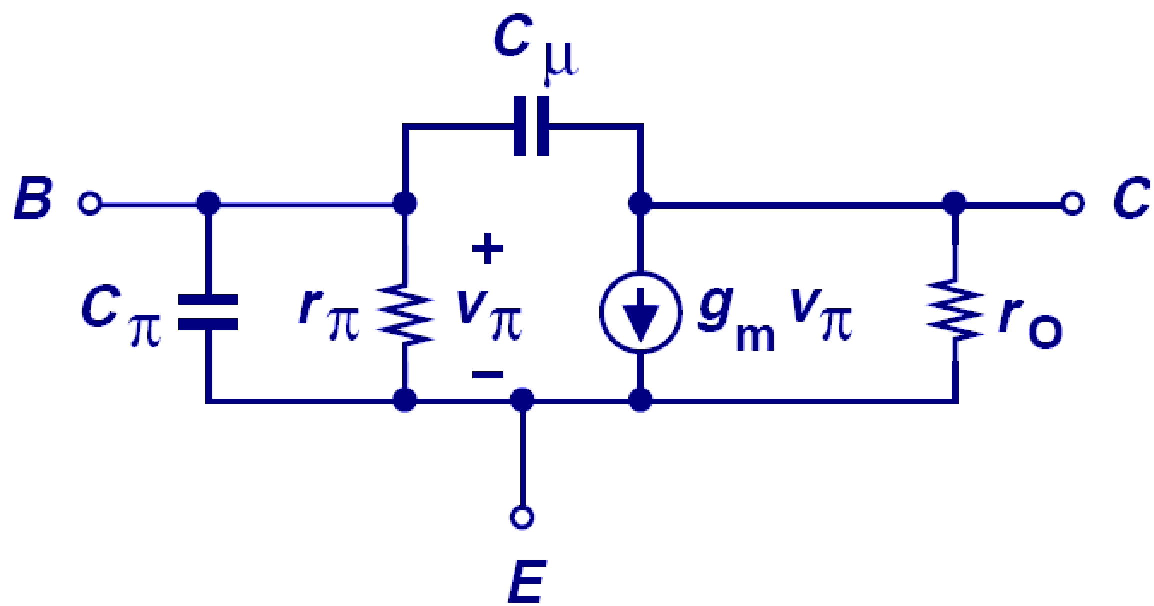
- CE/CS Stages
- CB/CG Stages
- Followers
- Cascode Stage
- Differential Pair

Typical Frequency Response



High-Frequency Bipolar Model

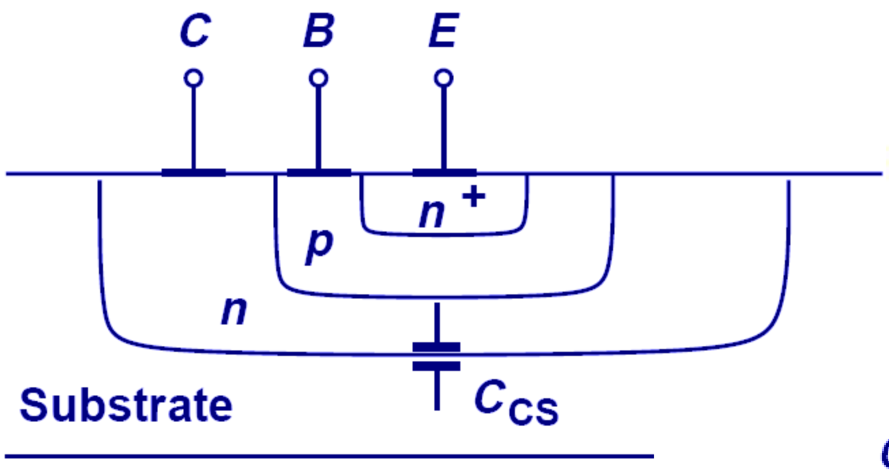


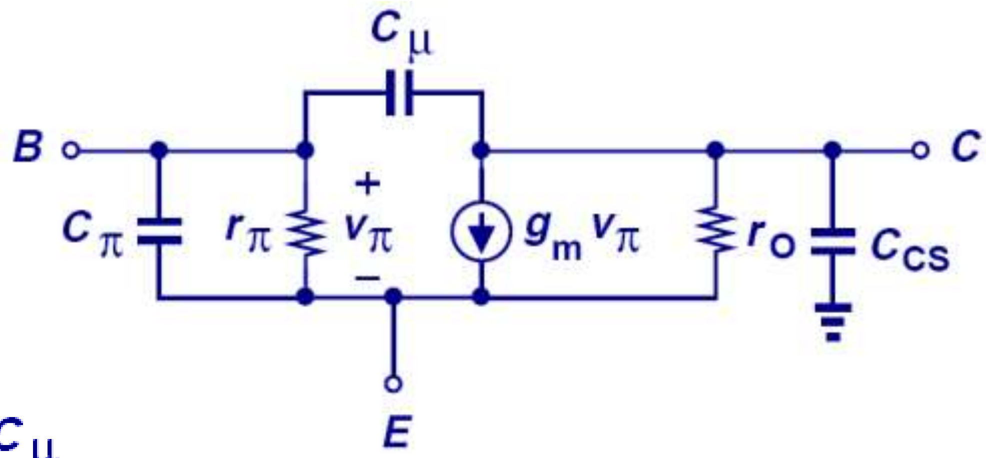


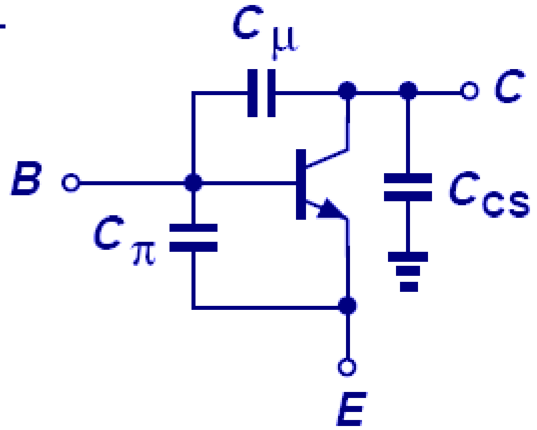
$$C_\pi = C_b + C_{je}$$

- **At high frequency, capacitive effects come into play. C_b represents the base charge, whereas C_μ and C_{je} are the junction capacitances.**

High-Frequency Model of Integrated Bipolar Transistor

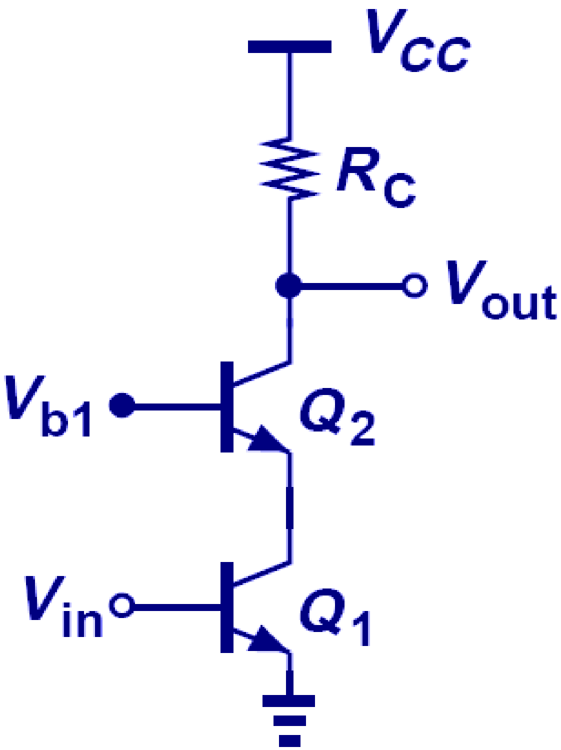


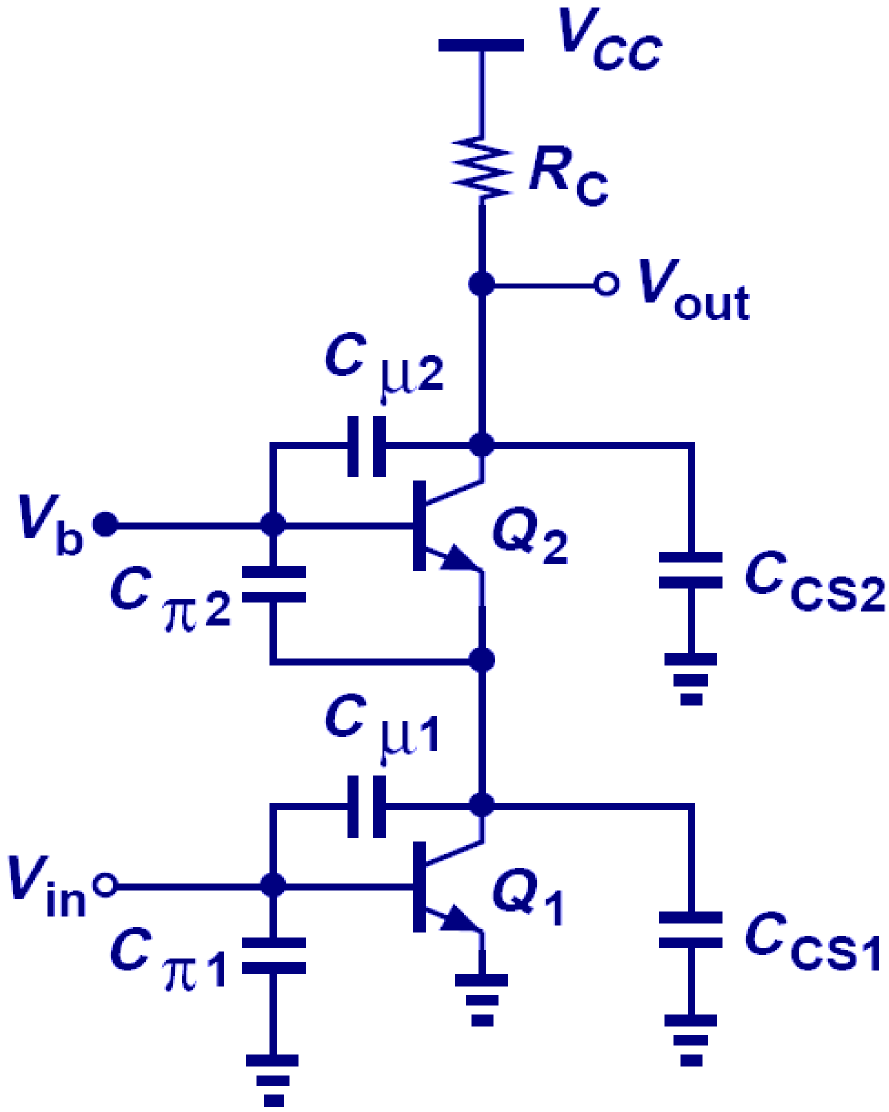




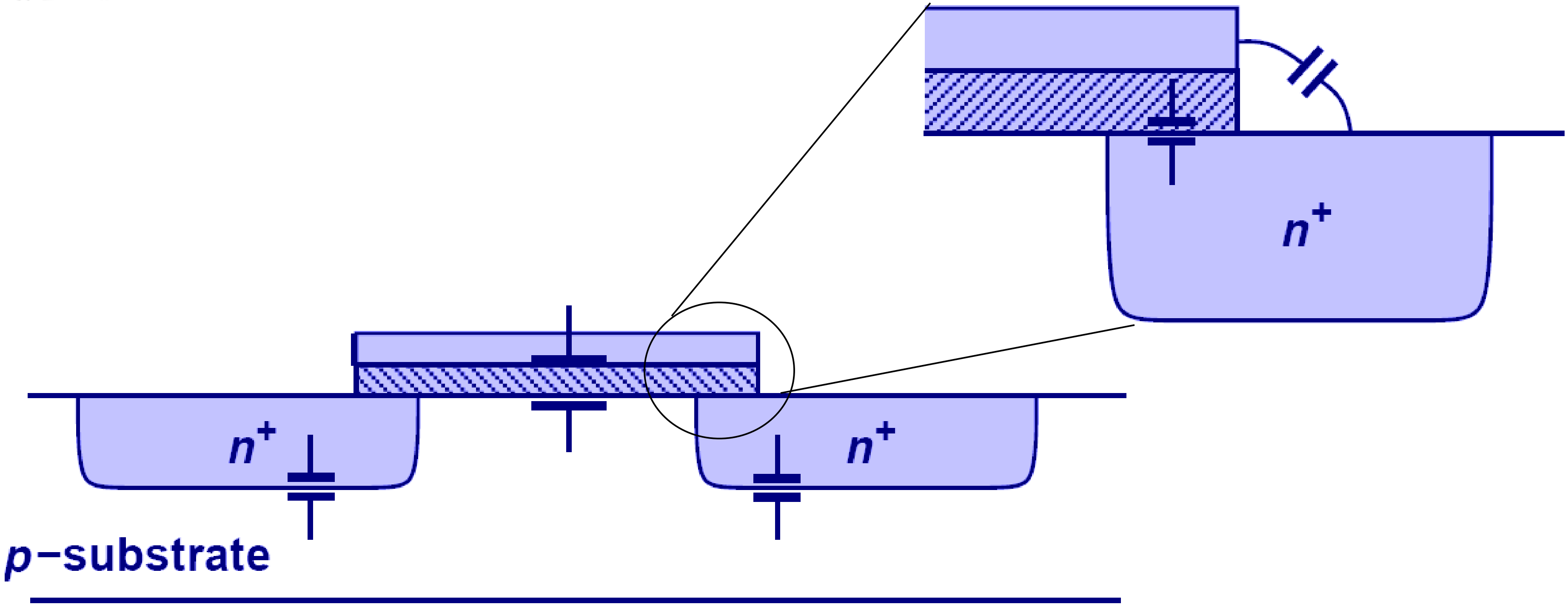
- **Since an integrated bipolar circuit is fabricated on top of a substrate, another junction capacitance exists between the collector and substrate, namely C_{CS}.**

Example: Capacitance Identification



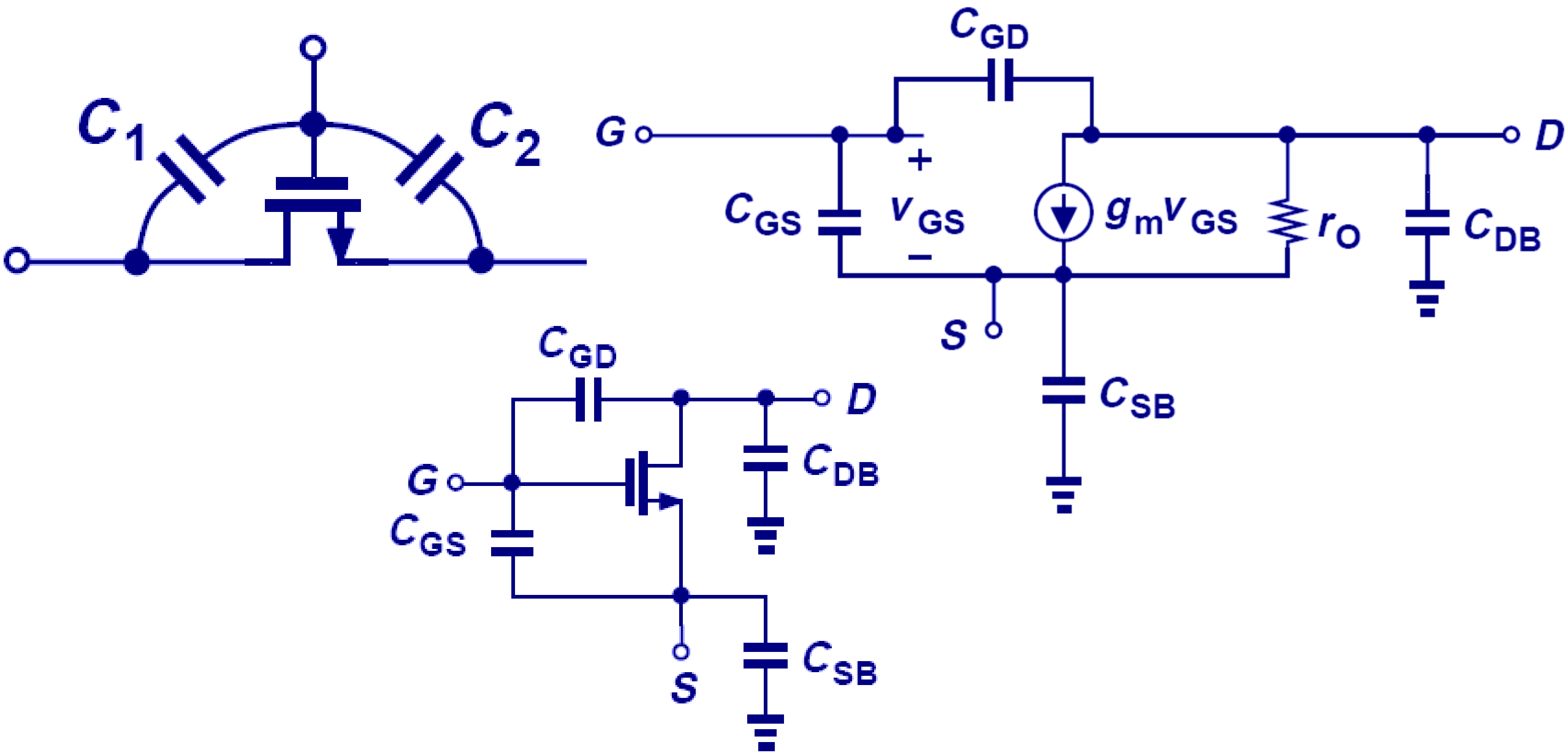


MOS Intrinsic Capacitances



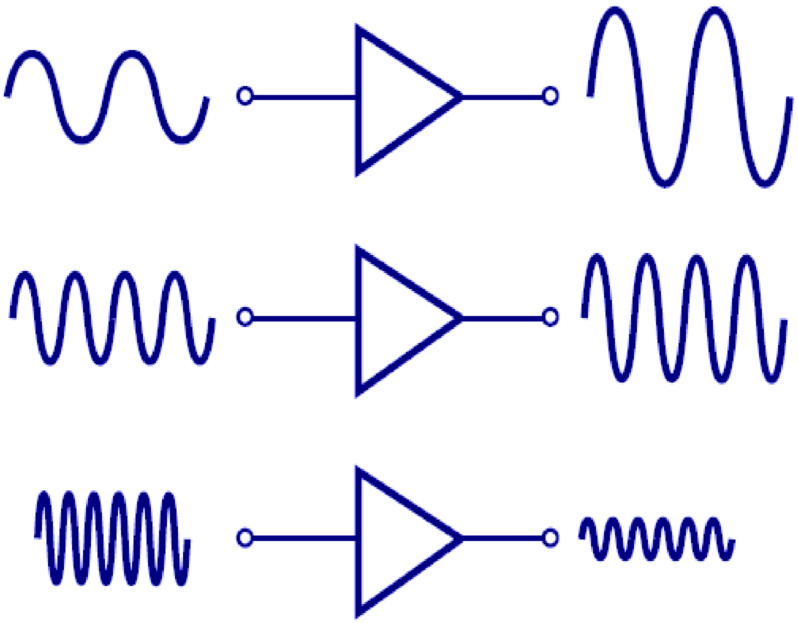
- For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.

Gate Oxide Capacitance Partition and Full Model



- The gate oxide capacitance is often partitioned between source and drain. In saturation, $C_2 \sim C_{gate}$, and $C_1 \sim 0$. They are in parallel with the overlap capacitance to form C_{GS} and C_{GD}.

High Frequency Roll-off of Amplifier



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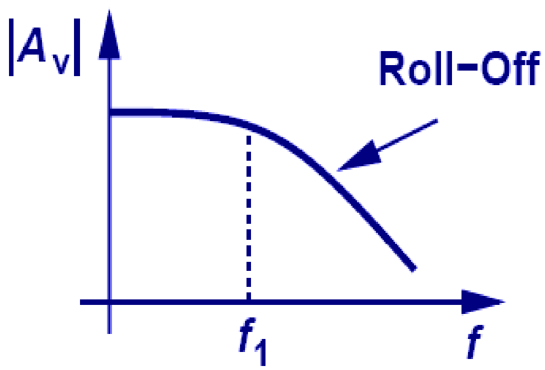
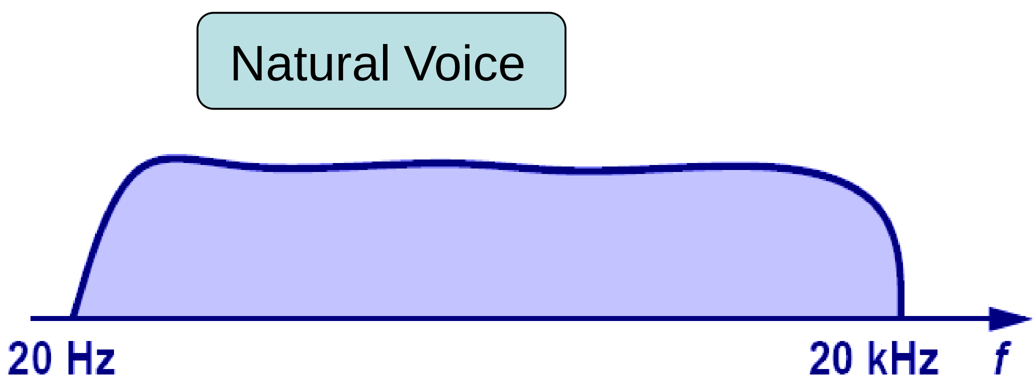


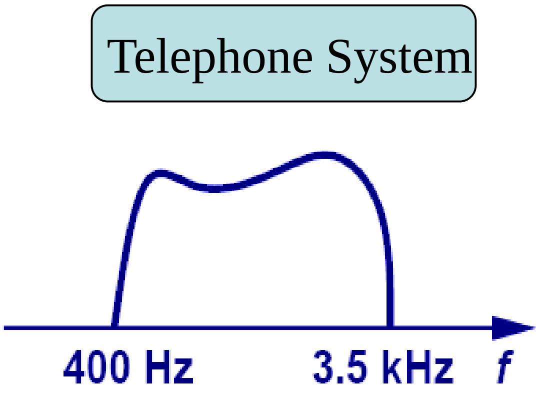
(b)

- **As frequency of operation increases, the gain of amplifier decreases. This chapter analyzes this problem.**

Example11.1: Human Voice I



(a)

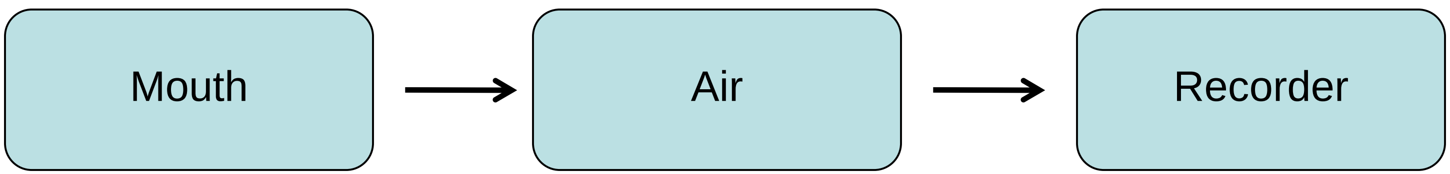


(b)

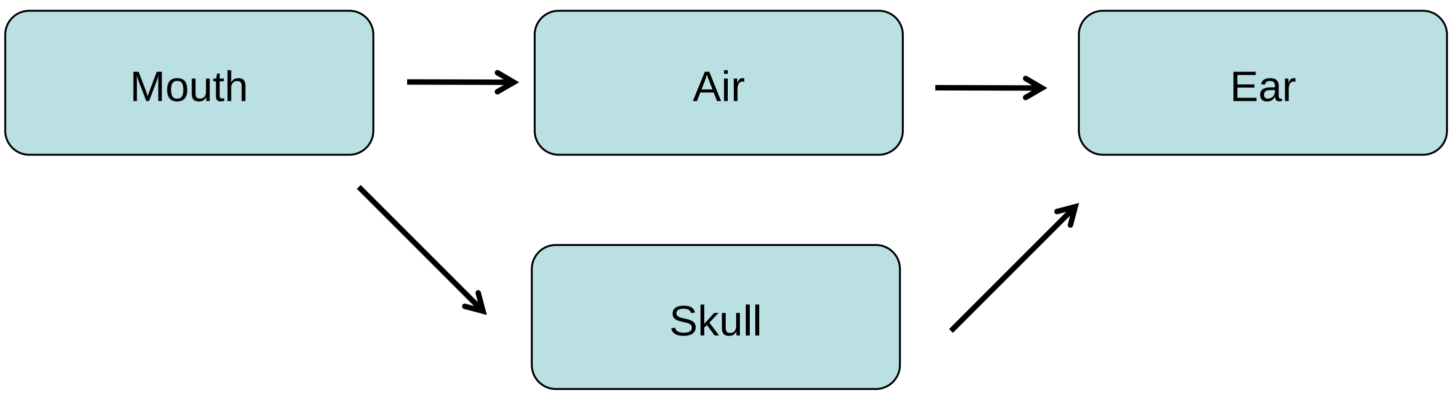
- **Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz.**
- **Therefore phone conversation differs from face-to-face conversation.**

Example 11.2: Human Voice II

Path traveled by the human voice to the voice recorder



Path traveled by the human voice to the human ear



- **Since the paths are different, the results will also be different.**

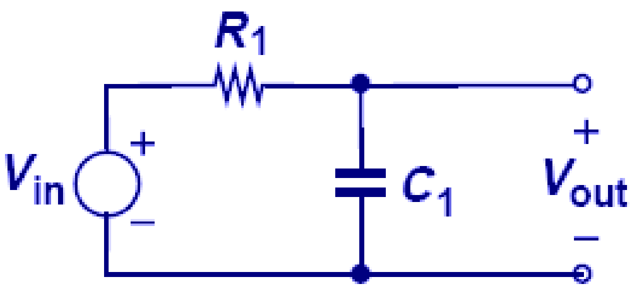
Example 11.3: Video Signal

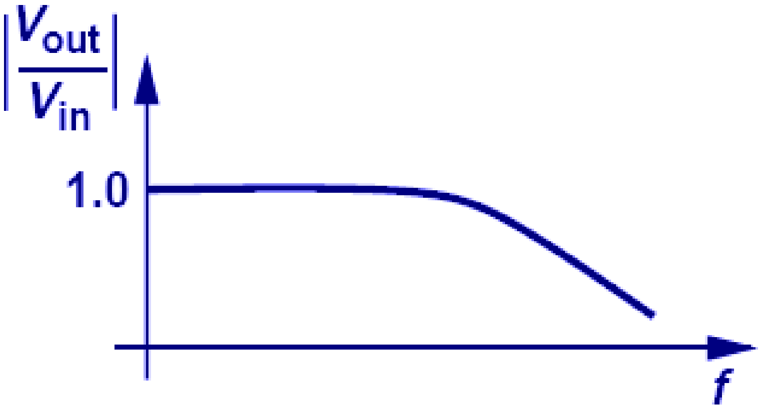
High Bandwidth

Low Bandwidth

- Video signals without sufficient **bandwidth** become fuzzy as they fail to abruptly change the **contrast of pictures** from complete **white** into complete **black**.

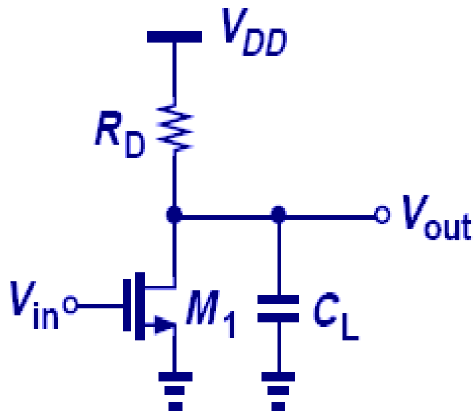
Gain Roll-off: Simple Low-pass Filter



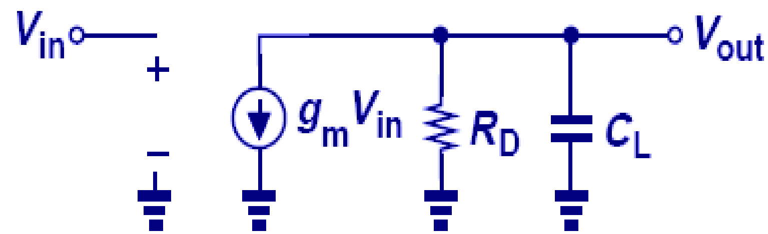


- **In this simple example, as frequency increases the impedance of C_1 decreases and**
- **the voltage divider consists of C_1 and R_1 attenuates V_{in} to a greater extent at the output.**

Example 11.4: Gain Roll-off: Common Source



(a)

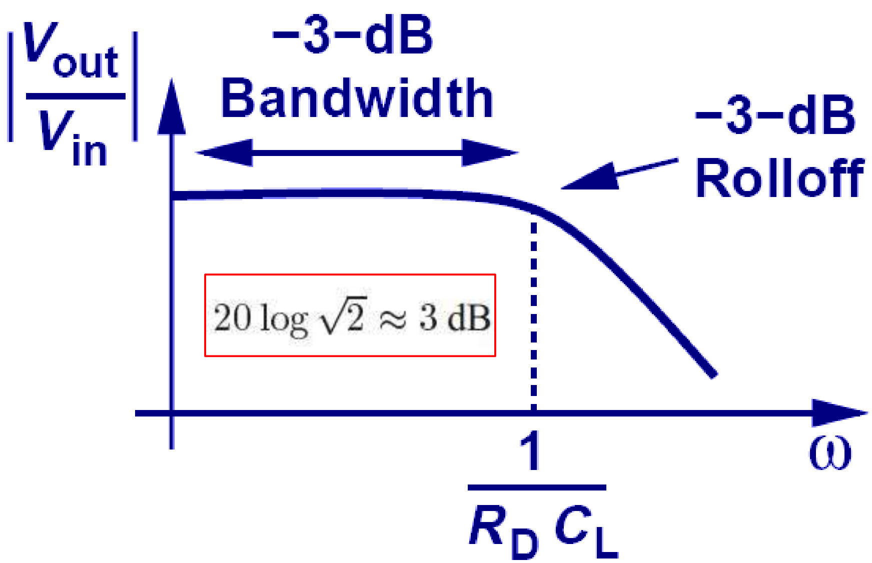


(b)

$$V_{out} = -g_m V_{in}\left(R_D \,\|\, \frac{1}{C_L s}\right) = \frac{-g_m R_D}{R_D C_L s + 1}.$$

- **The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will "steal" away some signal current and shunt it to ground.**

Frequency Response of the CS Stage

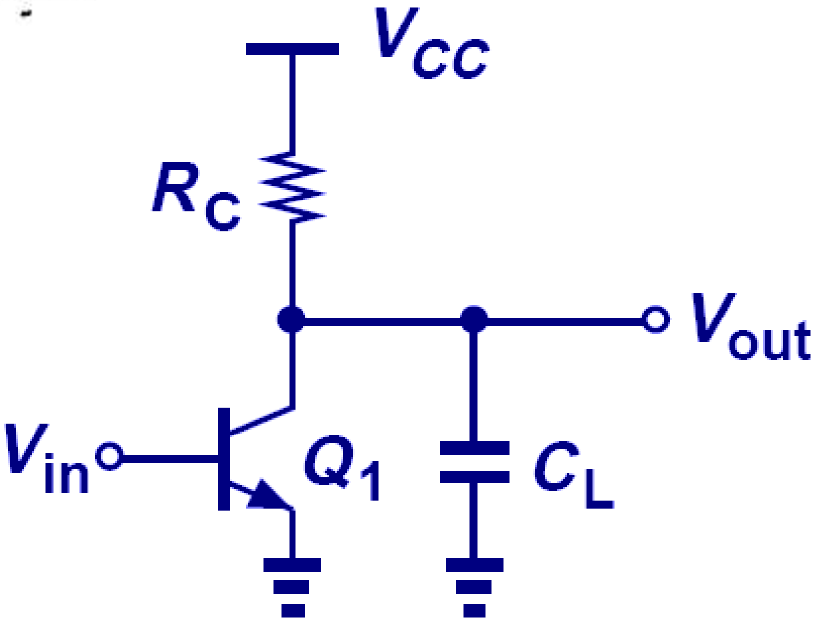


$$= \frac{-g_m R_D}{R_D C_L s + 1}.$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

- **At low frequency, the capacitor is effectively open and the gain is flat.**
- **As frequency increases, the capacitor tends to a short and the gain starts to decrease.**
- **A special frequency is $\omega = 1/(R_D C_L)$, where the gain drops by 3dB.**

Example11.5 : Figure of Merit

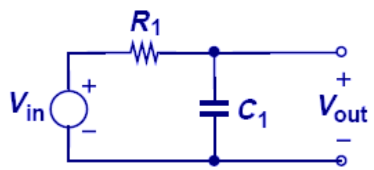


$$\frac{\text{Gain} \times \text{Bandwidth}}{\text{Power Consumption}} = \frac{\frac{I_C}{V_T} R_C \times \frac{1}{R_C C_L}}{I_C \cdot V_{CC}}$$

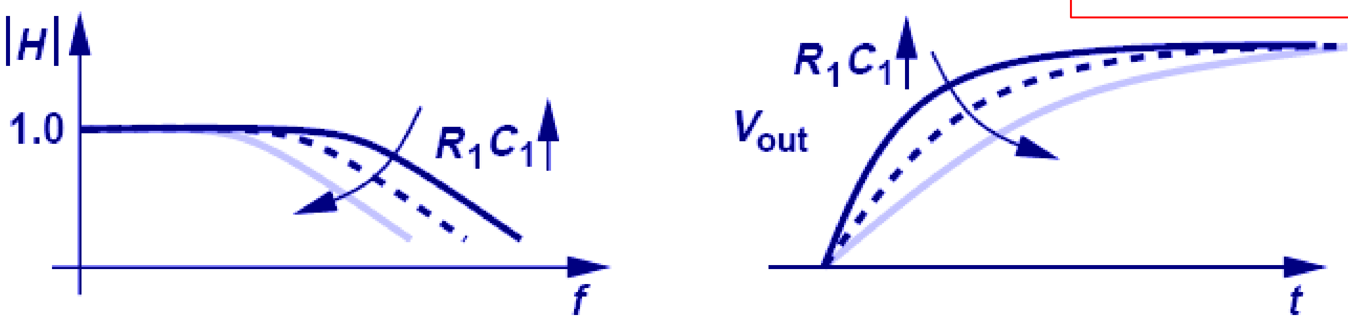
$$F.O.M. = \frac{1}{V_T V_{CC} C_L}$$

- **This metric quantifies a circuit's gain, bandwidth, and power dissipation.**
- **In the bipolar case, low temperature, supply, and load capacitance mark a superior figure of merit.**

Example 11.6: Relationship between Frequency Response and Step Response



$$H(s) = \frac{V_{out}}{V_{in}}(s) = \frac{\frac{1}{C_1 s}}{\frac{1}{C_1 s} + R_1} = \frac{1}{R_1 C_1 s + 1}.$$



(a) (b)

$$\left|H(s = j\omega)\right| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

$$V_{out}(t) = V_0 \left(1 - \exp\frac{-t}{R_1 C_1}\right) u(t)$$

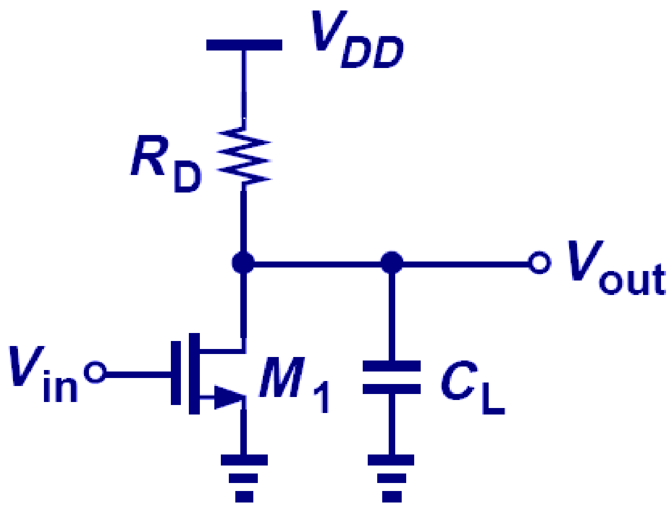
- **The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower.***

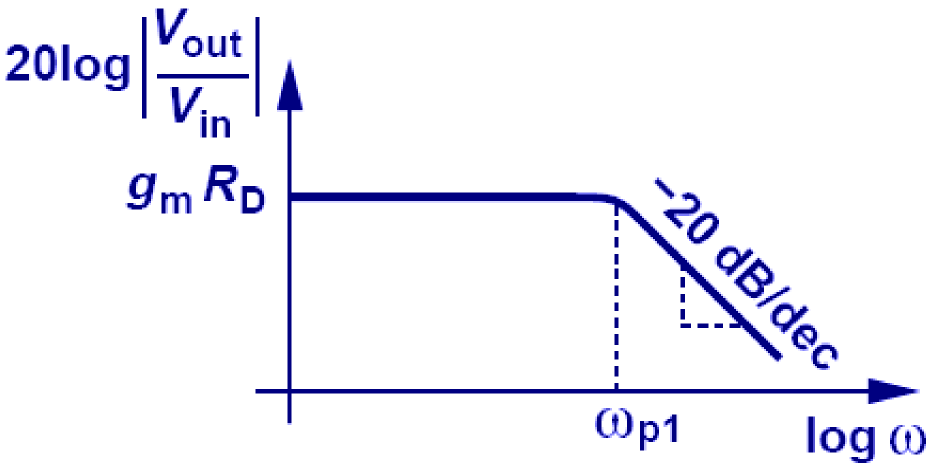
Bode Plot

$$H(s) = A_0 \frac{\left(1+\dfrac{s}{\omega_{z1}}\right)\left(1+\dfrac{s}{\omega_{z2}}\right)\cdots}{\left(1+\dfrac{s}{\omega_{p1}}\right)\left(1+\dfrac{s}{\omega_{p2}}\right)\cdots}$$

- **When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.**
- **When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec**

Example 11.7: Bode Plot

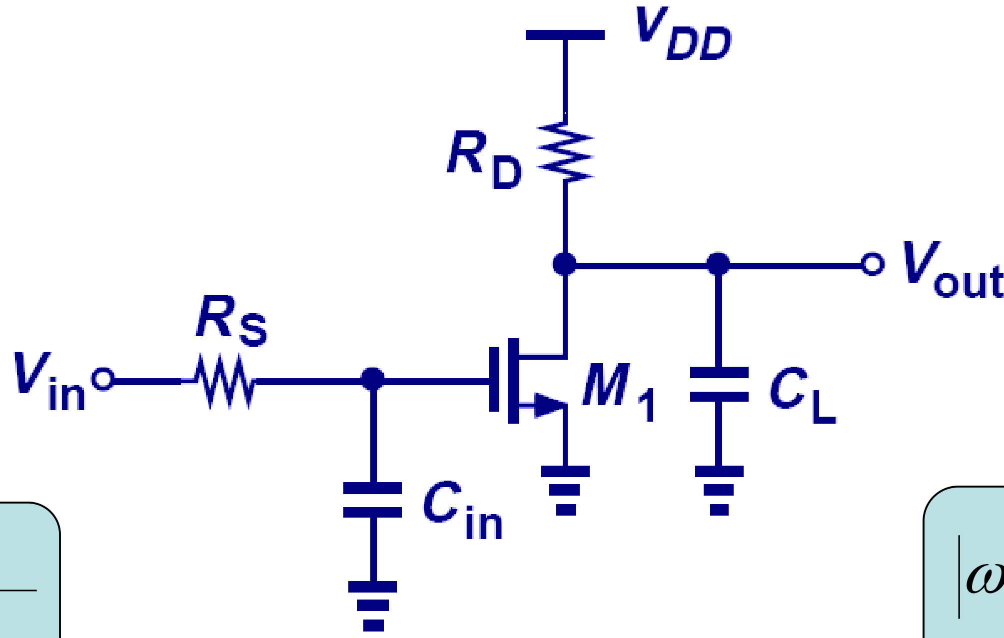




$$\left|\omega_{p1}\right| = \frac{1}{R_D C_L}$$

- The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}.

Pole Identification Example 11.8

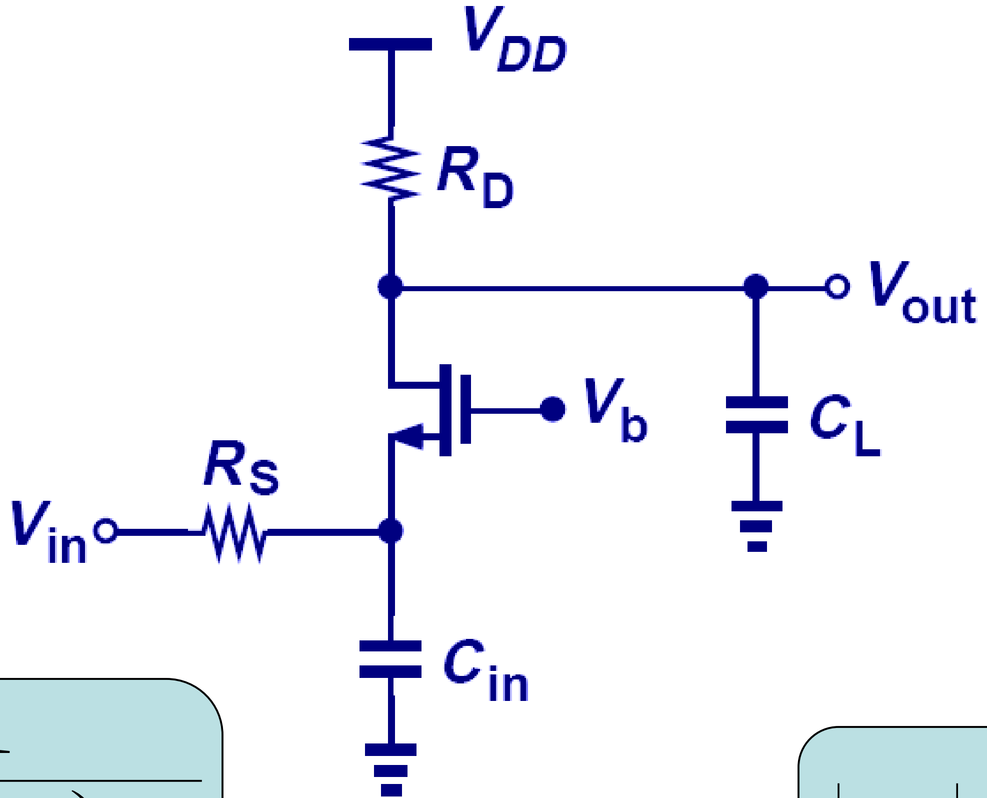


$$|\omega_{p1}| = \frac{1}{R_S C_{in}}$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

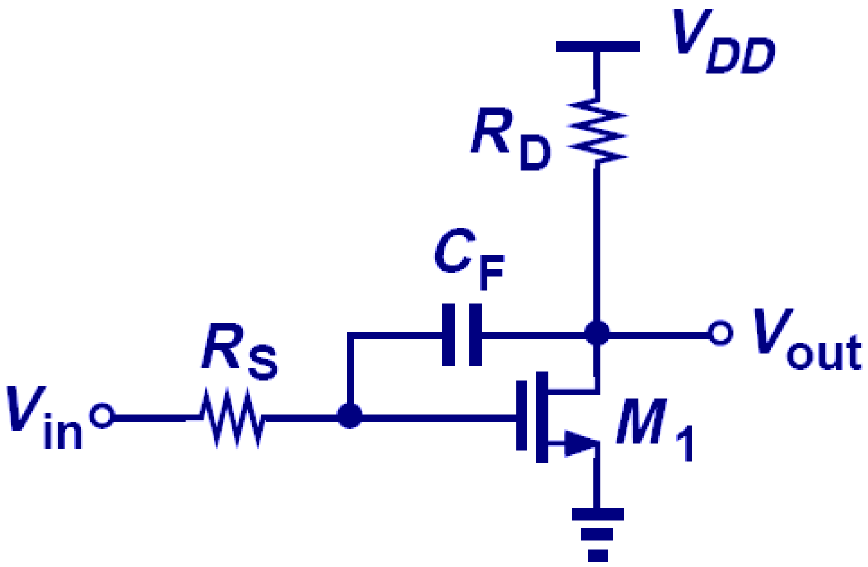
Pole Identification Example 11.9



$$|\omega_{p1}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_{in}}$$

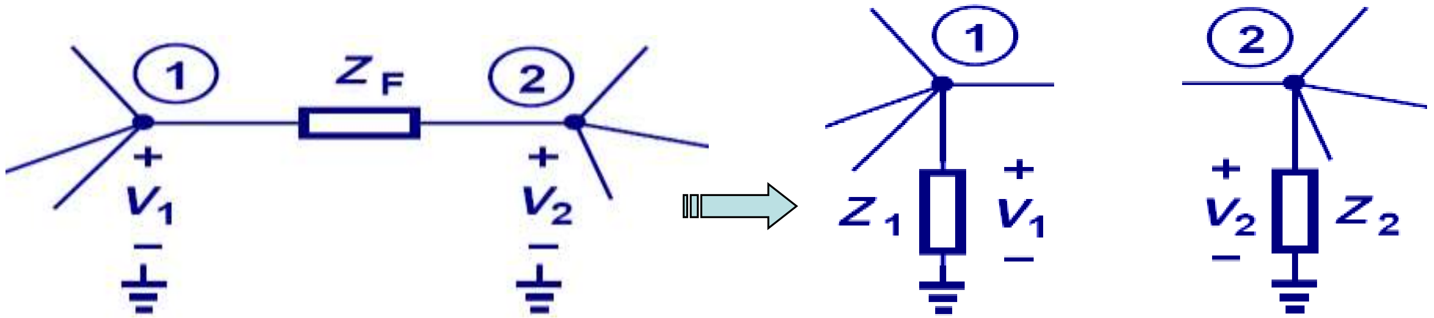
$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

Circuit with Floating Capacitor



- The **pole** of a circuit is computed by finding **the effective resistance and capacitance from a node to GROUND.**
- **The circuit above creates a problem since neither terminal of C_F is grounded.**

Miller's Theorem



$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}.$$

$$Z_1 = Z_F \frac{V_1}{V_1 - V_2}$$

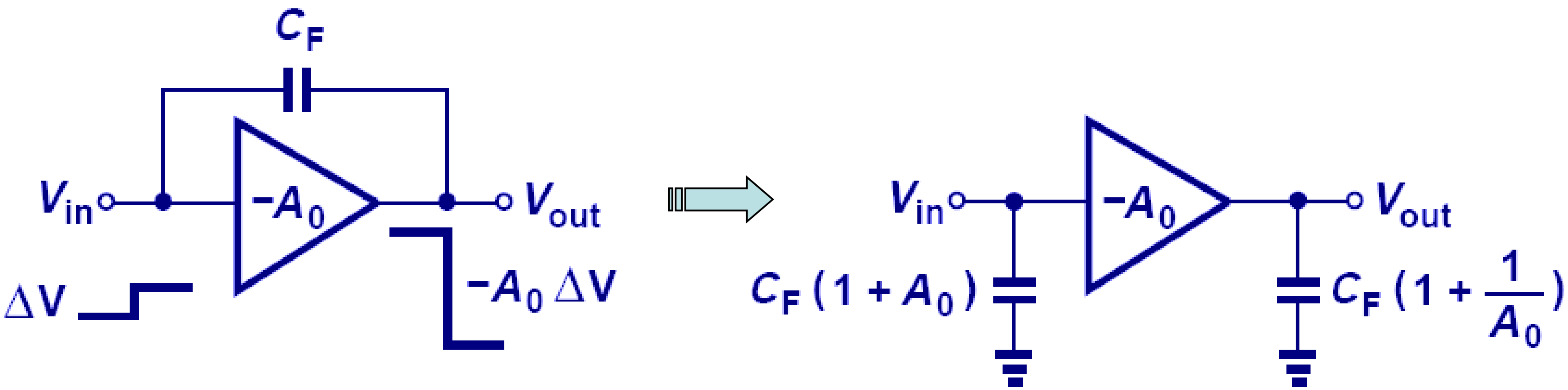
$$Z_2 = Z_F \frac{-V_2}{V_1 - V_2}$$

$$Z_1 = \frac{Z_F}{1 - A_v}$$

$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

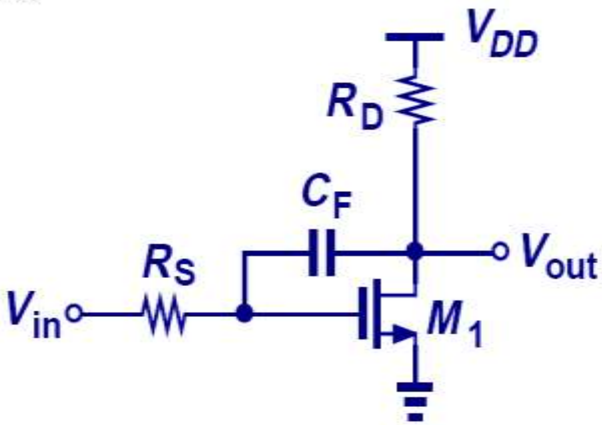
- **If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.**

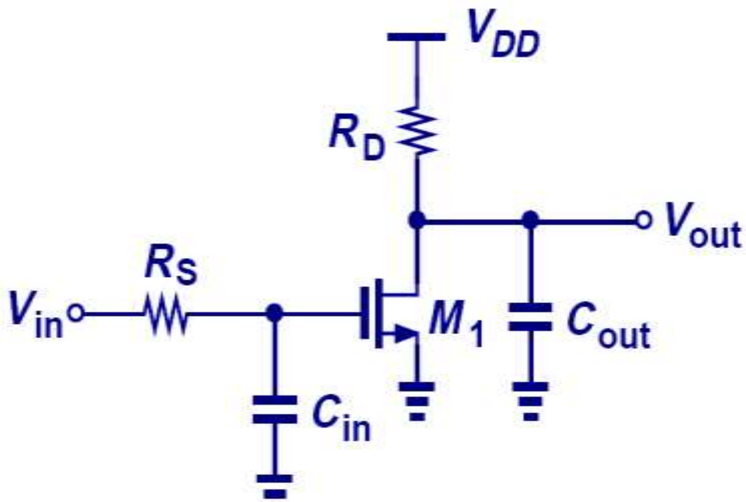
Miller Multiplication



- With **Miller's** theorem, we can separate the **floating** capacitor.
- However, the input capacitor is **larger** than the original floating capacitor.
- We call this **Miller multiplication**.

Example 11.10: Miller Theorem





$$C_{in} = (1 + A_0)C_F = (1 + g_m R_D)C_F$$

$$\omega_{in} = \frac{1}{R_S C_{in}}$$

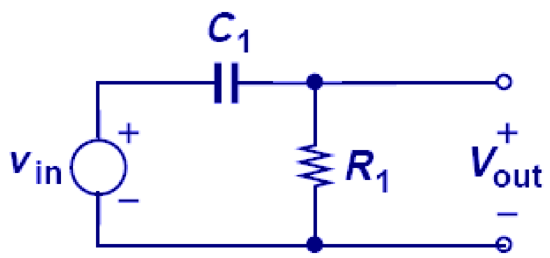
$$C_{out} = \left(1 + \frac{1}{g_m R_D}\right) C_F$$

$$\omega_{out} = \frac{1}{R_D C_{out}}$$

$$\omega_{in} = \frac{1}{R_S (1 + g_m R_D) C_F}$$

$$\omega_{out} = \frac{1}{R_D \left(1 + \dfrac{1}{g_m R_D}\right) C_F}$$

High-Pass Filter Response



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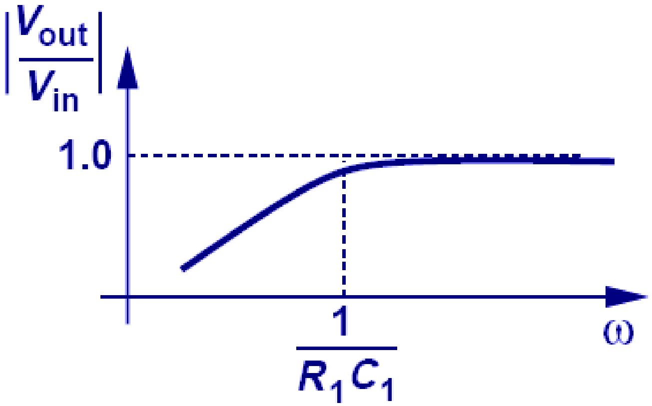


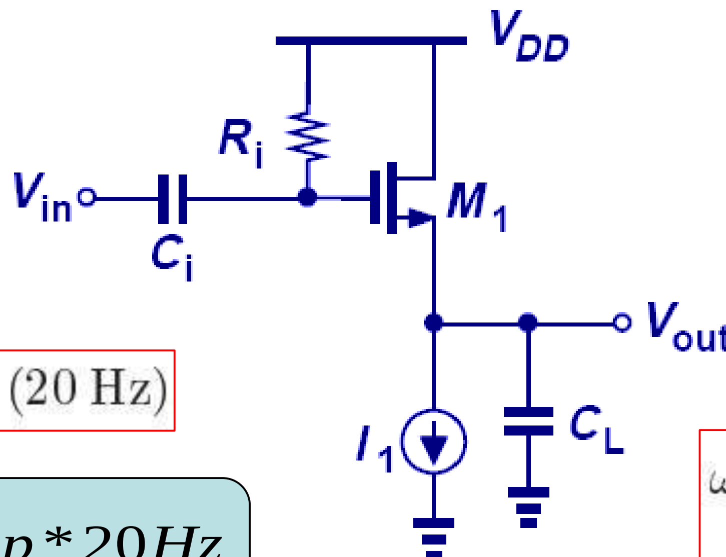
(b)

$$\frac{V_{out}}{V_{in}}(s) = \frac{R_1}{R_1 + \frac{1}{C_1 s}} = \frac{R_1 C_1 s}{R_1 C_1 s + 1},$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1 C_1 \omega}{\sqrt{R_1^2 C_1^2 \omega_1^2 + 1}}$$

- **The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.**

Example 11.11: Audio Amplifier



$$R_i = 100K\Omega$$
$$g_m = 1/200\Omega$$

$1/(R_iC_i)$ to $2\pi \times (20\text{ Hz})$

$$|\omega_{p1}| = \frac{1}{R_{in}C_{in}} = 2p * 20Hz$$

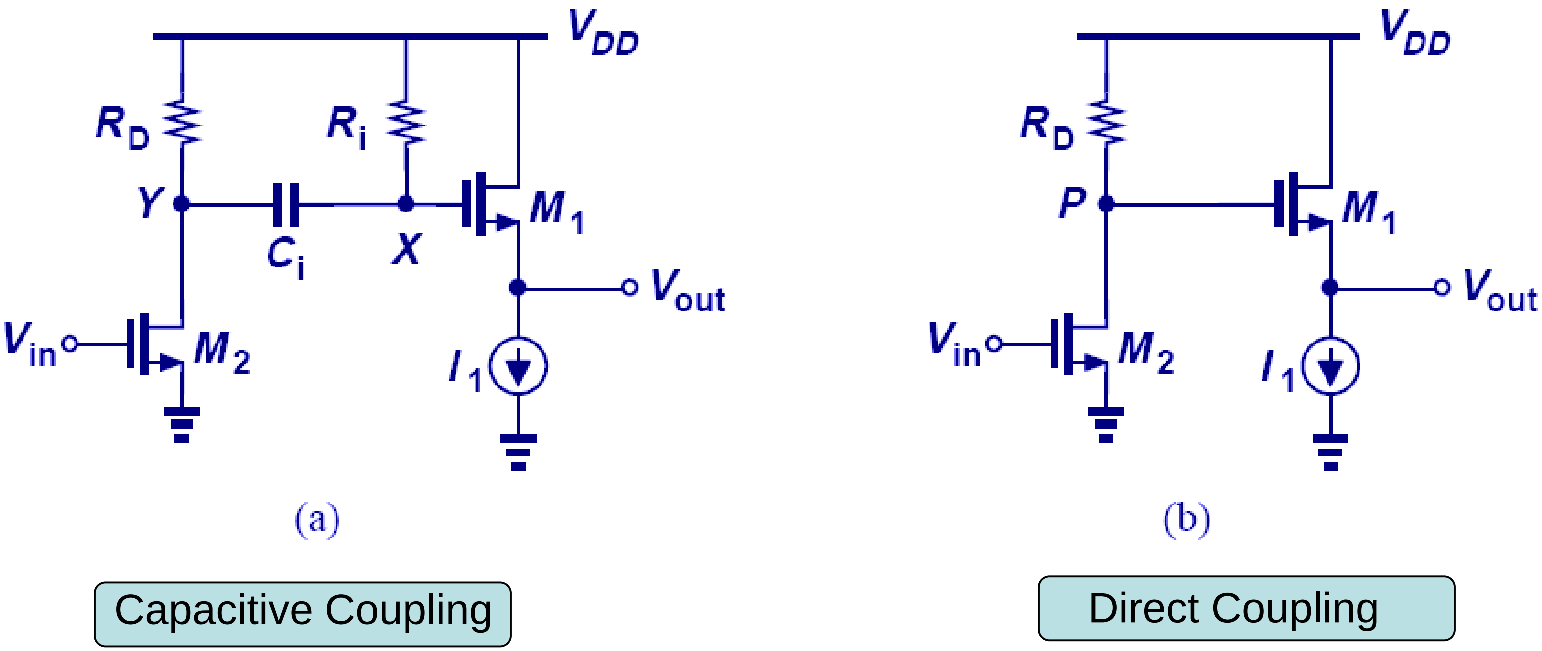
$$C_i = 79.6nF$$

$$\omega_{p,out} = \frac{g_m}{C_L} = 2\pi \times (20\text{ kHz}),$$

$$C_L = 39.8nF$$

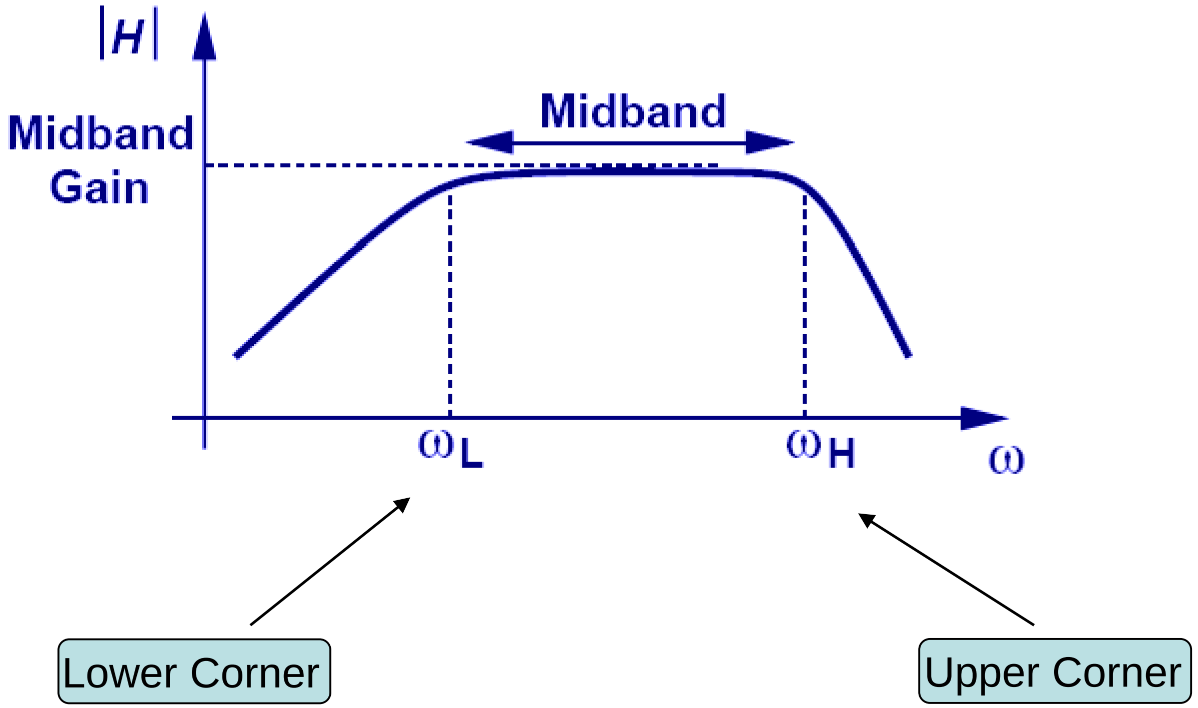
- **In order to successfully pass audio band frequencies (20 Hz-20 KHz), large input and output capacitances are needed.**

Capacitive Coupling vs. Direct Coupling

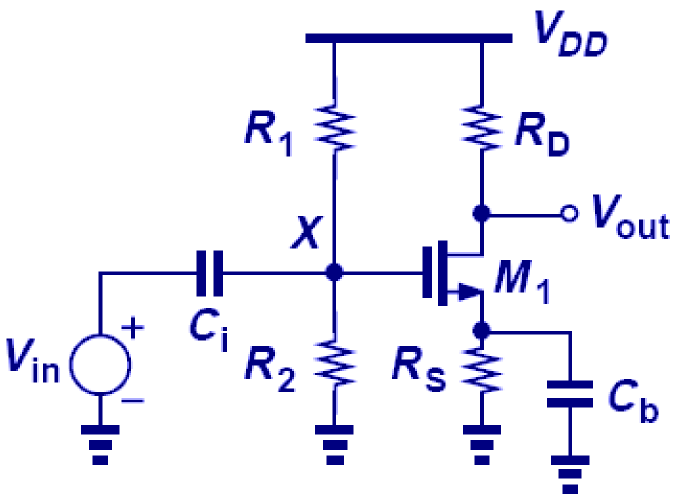


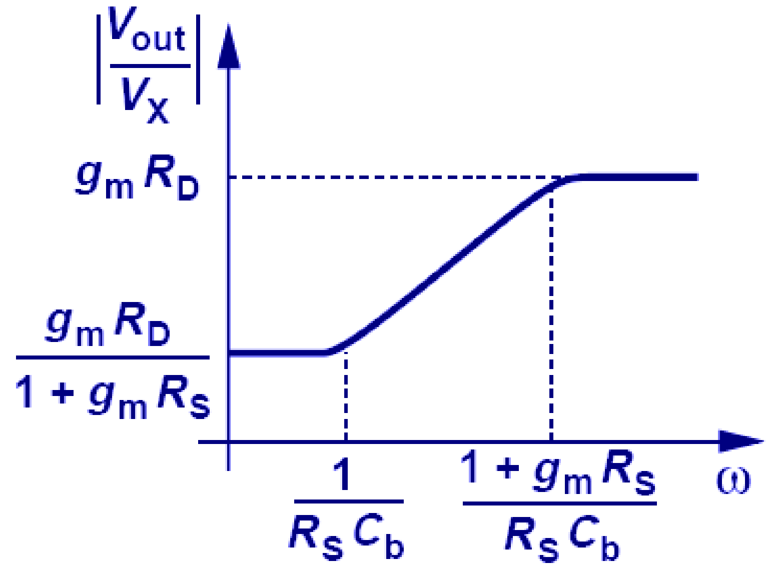
- **Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.**
- **This technique allows independent bias conditions between stages. Direct coupling does not.**

Typical Frequency Response



Frequency Response of CS Stage with Bypassed Degeneration



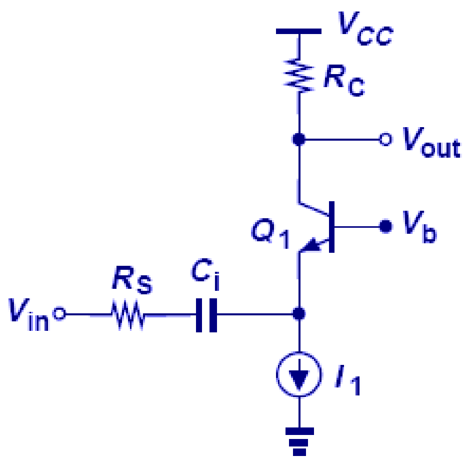


$$\frac{V_X}{V_{in}}(s) = \frac{R_1||R_2}{R_1||R_2 + \frac{1}{C_i s}}$$

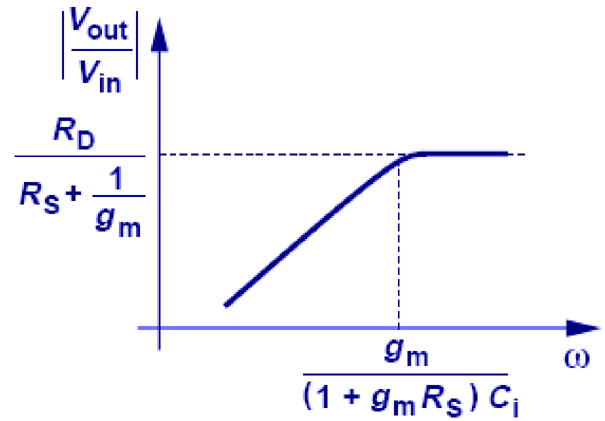
$$\left|\frac{V_{out}}{V_X}(s)\right| = \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

- **In order to increase the midband gain, a capacitor C_b is placed in parallel with R_s.**
- **The pole frequency must be well below the lowest signal frequency to avoid the effect of degeneration.**

Low Frequency Response of CB and CG Stages



(a)



(b)

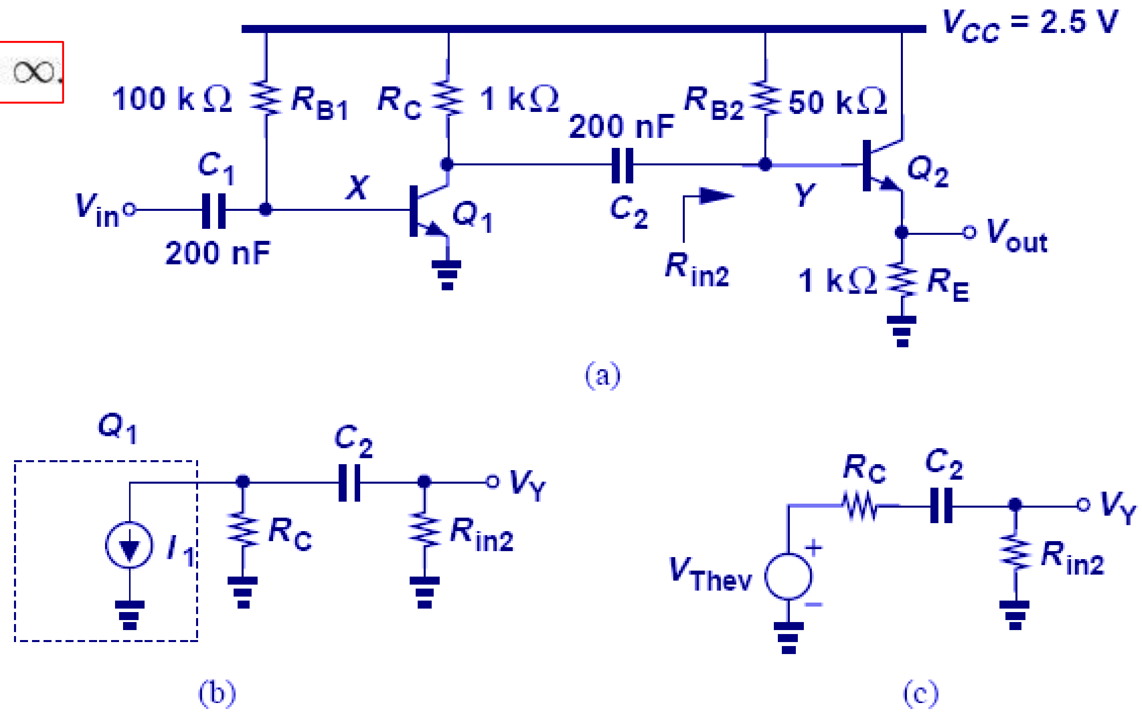
$$\frac{V_{out}}{V_{in}}(s) = \frac{R_C}{R_S + (C_i s)^{-1} + 1/g_m}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m R_C C_i s}{(1+g_m R_S)C_i s + g_m}$$

- **As with CE and CS stages, the use of capacitive coupling leads to low-frequency roll-off in CB and CG stages (although a CB stage is shown above, a CG stage is similar).**

Example 11.28: Capacitive Coupling

$I_S = 5 \times 10^{-16}$ A, $\beta = 100$, and $V_A = \infty$.



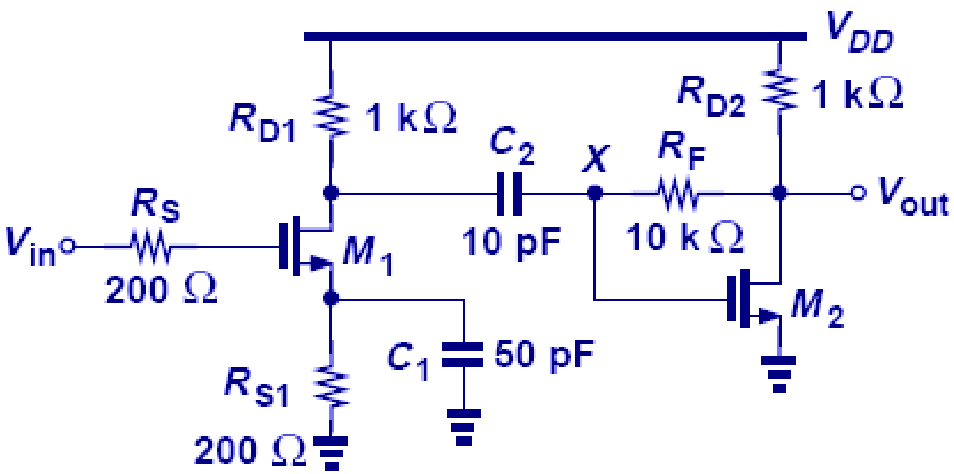
$$R_{in2} = R_{B2} \parallel \left[r_{\pi 2} + (\beta + 1) R_E \right]$$

$$\omega_{L1} = \frac{1}{(r_{\pi 1} \parallel R_{B1}) C_1} = 2\pi \times (542 Hz)$$

$$\omega_{L2} = \frac{1}{(R_C + R_{in2}) C_2} = \pi \times (22.9 Hz)$$

33

Example 11.29:
IC Amplifier – Low Frequency Design

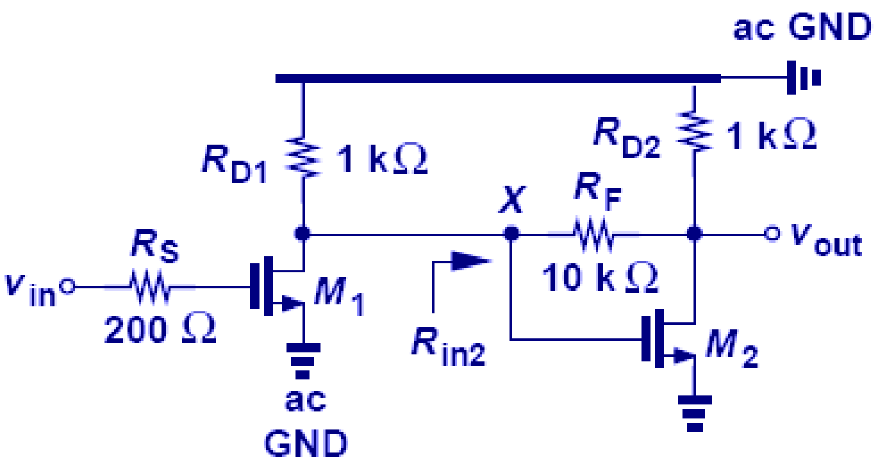


$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

$$\omega_{L1} = \frac{g_{m1}R_{S1} + 1}{R_{S1}C_1} = 2\pi \times (42.4MHz)$$

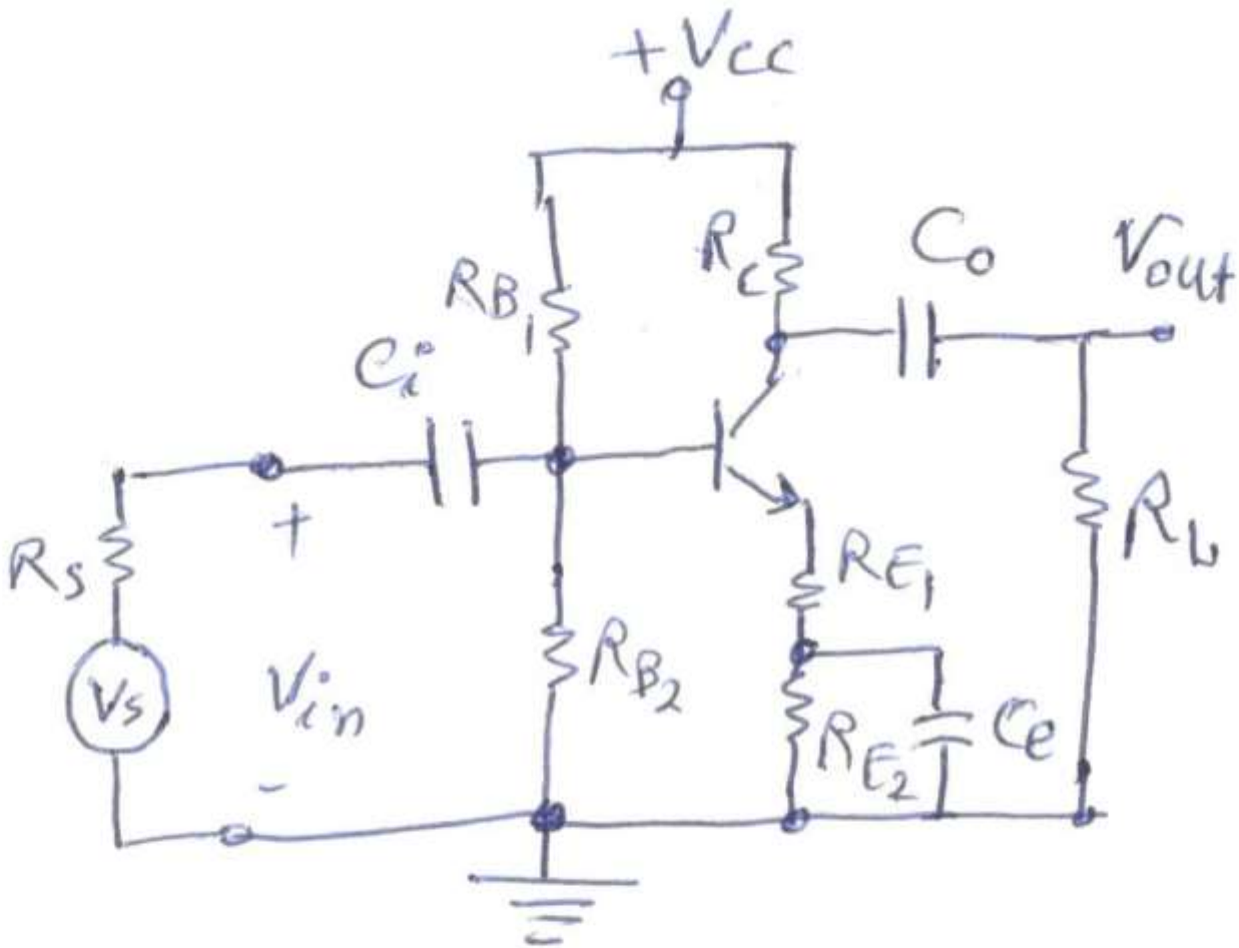
$$\omega_{L2} = \frac{1}{(R_{D1} + R_{in2})C_2} = 2\pi \times (6.92MHz)$$

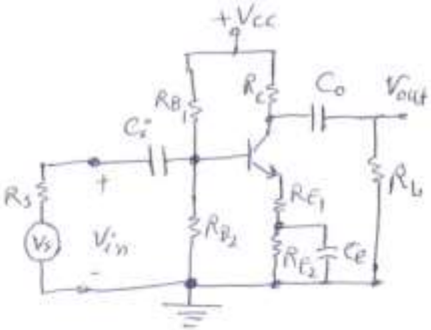
Example: IC Amplifier – Midband Design



$$\frac{v_X}{v_{in}} = -g_{m1}\left(R_{D1} \parallel R_{in2}\right) = -3.77$$

Example C.E:Design





تعیین C_i از روی f_L: $f_L = \dfrac{1}{2\pi R_{th(i)} \cdot C_i}$, $C_i = \dfrac{1}{2\pi f_L \cdot R_{th(i)}}$ (اولیه)

$$R_{th(i)} = R_S + \left[(R_{B_1} \| R_{B_2}) \| \left(r_\pi + R_{E_1}(\beta+1) \right) \right]$$

تعیین C_o از روی f_L: $f_L = \dfrac{1}{2\pi R_{th(o)} \cdot C_o}$, $C_o = \dfrac{1}{2\pi f_L \cdot R_{th(o)}}$ (اولیه)

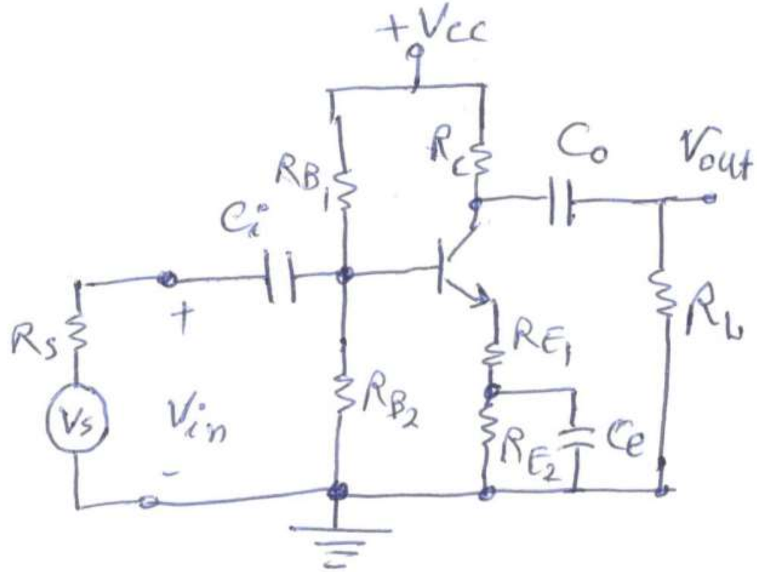
$$R_{th(o)} \approx R_L + R_C$$

یا (دقیق‌تر): $R_{th(o)} = R_L + \left\{ R_C \| \left(r_o + (1+g_m r_o)\left[R_{E_1} \| (r_\pi + R_{BB}) \right] \right) \right\}$

تعیین C_e از روی f_L: $f_L = \dfrac{1}{2\pi R_{th(e)} \cdot C_e}$, $C_e = \dfrac{1}{2\pi f_L \cdot R_{th(e)}}$

$$R_{th(e)} \simeq R_{E_2} \| \left(R_{E_1} + \frac{1}{g_m} \right)$$

یا $R_{th(e)} \simeq R_{E_2} \| \left[R_{E_1} + \dfrac{R_{BB} + r_\pi}{\beta+1} \right]$



معمولاً کوچکترین مقدار اولیه خازنهای C_i و C_o را که از طریق فوق محاسبه می‌شود، در ده ضرب می‌کنیم و عملاً C_i و C_o واقعی در مدار را ده برابر بزرگتر انتخاب می‌نماییم تا f_L، ده برابر کوچکتر شود و f_L فقط ناشی از خازن بزرگتر که همان C_e باشد، تعیین می‌شود.