



# Fundamentals of Microelectronics II

## فصل ۱۰ - تقویت کننده های تفاضلی

(قسمت دوم - زوج تفاضلی ماسفتی)

**Mohammad Ali Mansouri- Birjandi**

**Majid Ghadrhan**

**Faculty of Electrical and Computer Engineering  
University of Sistan and Baluchestan (USB)**

**[mansouri@ece.usb.ac.ir](mailto:mansouri@ece.usb.ac.ir) ,  
[mamansouri@yahoo.com](mailto:mamansouri@yahoo.com)**

## Chapter 10 Differential Amplifiers

- **10.3 MOS Differential Pair**
- **10.4 Cascode Differential Amplifiers**
- **10.5 Common-Mode Rejection**
- **10.6 Differential Pair with Active Load**



## یادآوری ۱-۱: تعریف سیگنال تفاضلی و مشترک

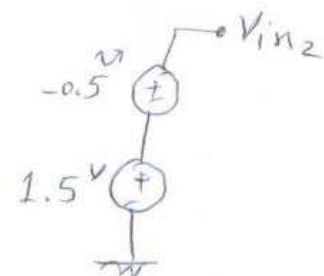
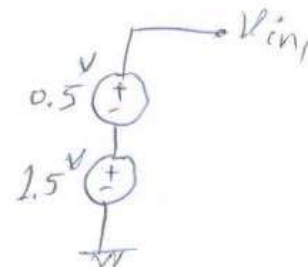
$$\begin{cases} V_{id} \triangleq V_{in1} - V_{in2} \\ V_{cm} \triangleq \frac{1}{2} (V_{in1} + V_{in2}) \end{cases}$$

$$\begin{cases} V_{in1} = V_{cm} + \frac{1}{2} V_{id} \\ V_{in2} = V_{cm} - \frac{1}{2} V_{id} \end{cases}$$

۱<sup>دا</sup>ا:

$$V_{in1} = 2V, \quad V_{in2} = 1V$$

$$\Rightarrow V_{cm} = \frac{2+1}{2} = 1.5V \Rightarrow \begin{cases} V_{id} = 2 - 1 = 1V \end{cases}$$





$$V_{out} = A_d (V_{id}) + A_{cm} V_{cm}$$

با استفاده از جمع و تفریق

$$\left\{ \begin{array}{l} V_{id} \triangleq V_{in} = V_{in1} - V_{in2} \\ V_{cm} \triangleq \frac{1}{2} (V_{in1} + V_{in2}) \end{array} \right.$$

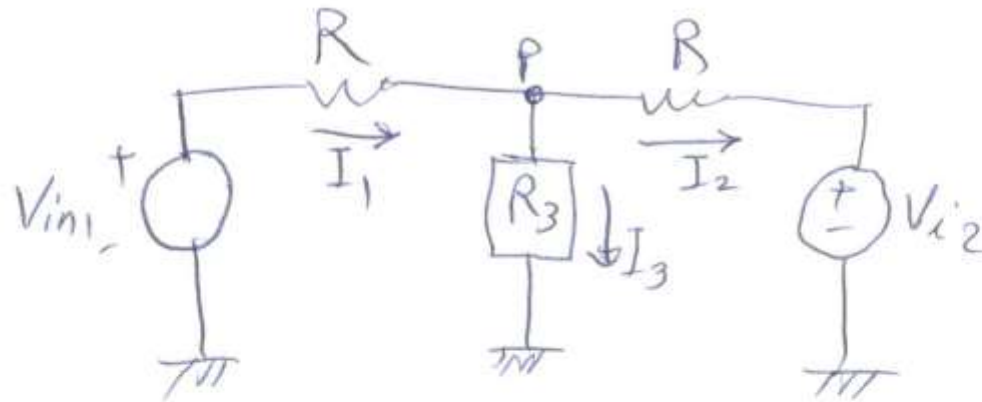
$$\left\{ \begin{array}{l} V_{id} \triangleq V_{in} = V_{in1} - V_{in2} \\ V_{cm} \triangleq \frac{1}{2} (V_{in1} + V_{in2}) \end{array} \right.$$

برای حالت ایده‌آل (توازن کامل و ...):  $A_{cm} = 0$

حذف مد مشترک در تقریب کشنده تفاضلی

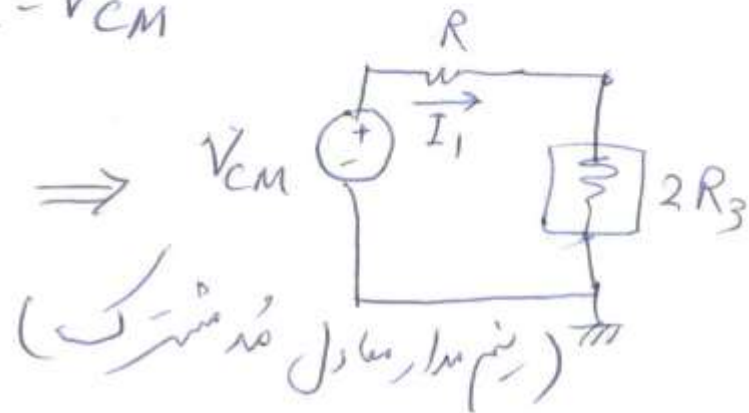


## یادآوری ۲: نیم مدار حالت مشترک



//  
اگر:  $V_{in1} = V_{in2} = V_{CM}$

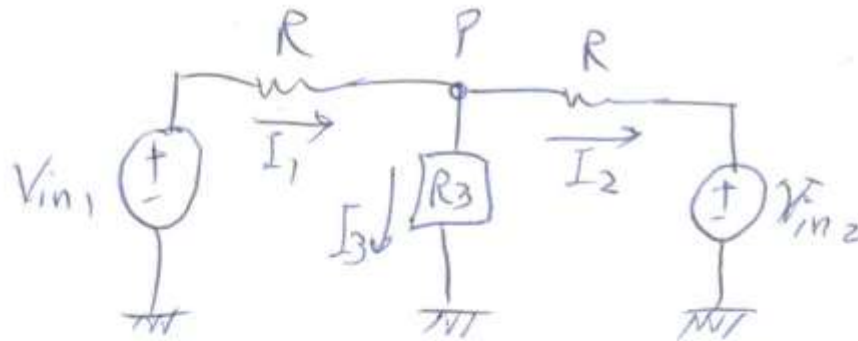
$$\Rightarrow \begin{cases} I_2 = -I_1 \\ I_3 = 2I_1 \end{cases}$$





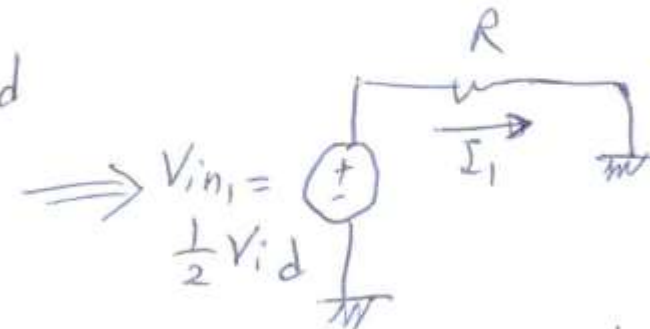
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## یادآوری ۳: نیم مدار حالت تفاضلی



$$\begin{cases} V_{in2} = -V_{in1} = -\frac{1}{2} V_{id} \\ V_{in1} = \frac{1}{2} V_{id} \end{cases}$$

$$\Rightarrow \begin{cases} I_2 = I_1 \\ I_3 = 0, \quad V_P = 0 \end{cases} \text{ (نیم مدار تفاضلی)}$$



نیم مدار تفاضلی  
(مدر تفاضلی)



## Differential Amplifiers

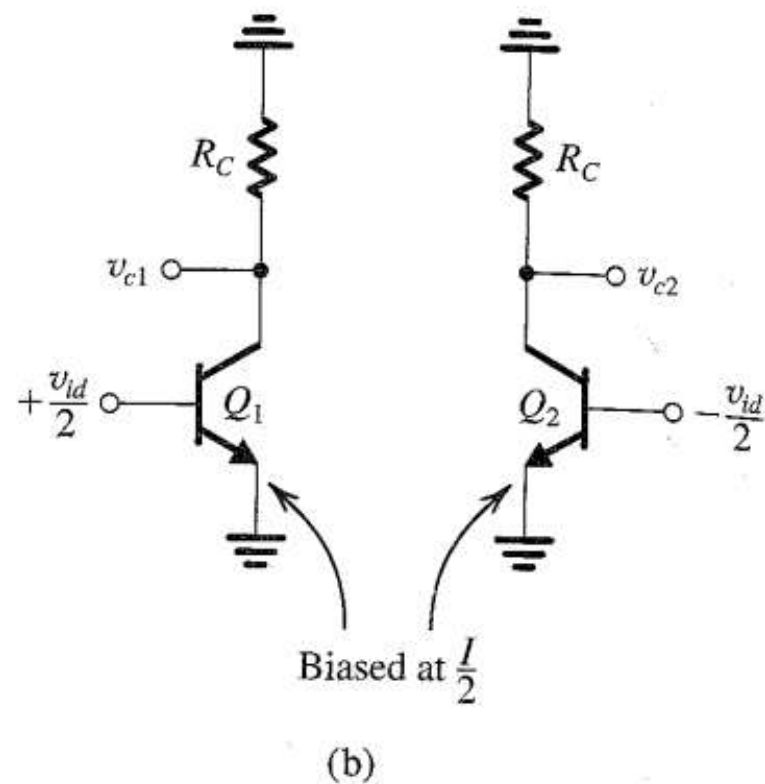
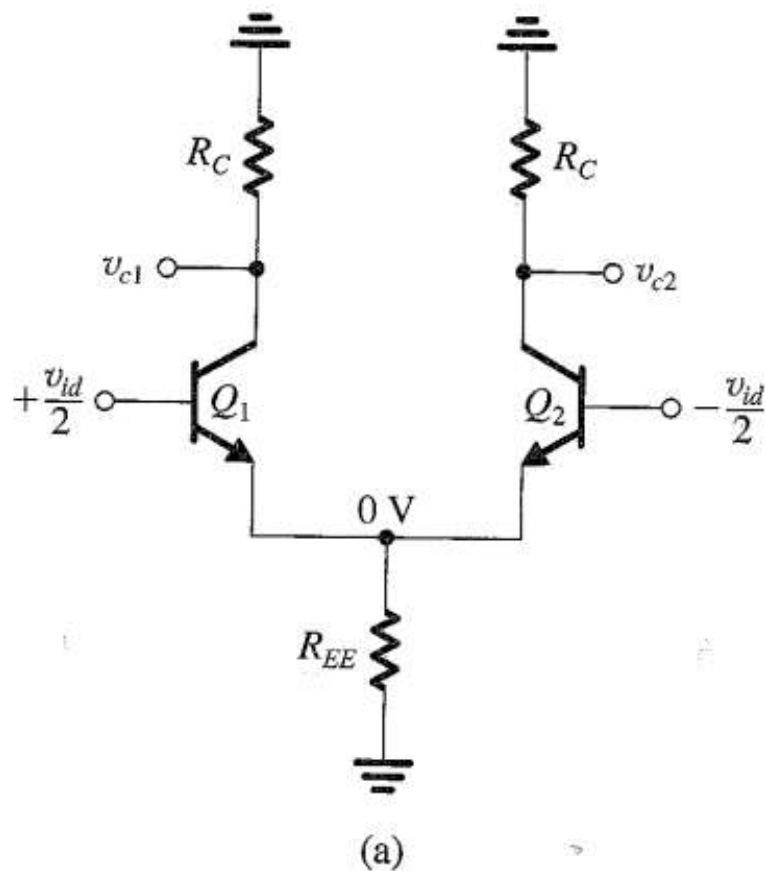
$$v_{icm} \equiv \frac{v_1 + v_2}{2}$$

$$v_{id} \equiv v_1 - v_2$$

$$v_o = A_d(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

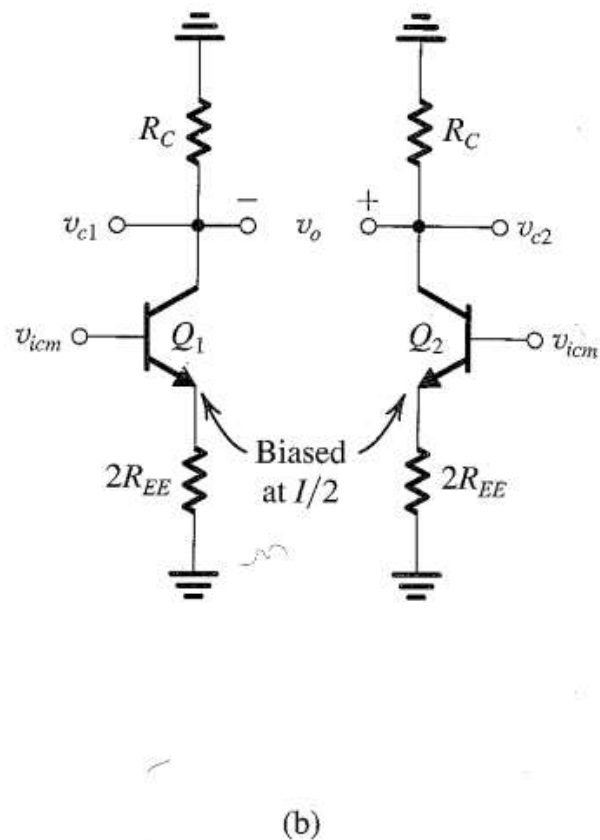
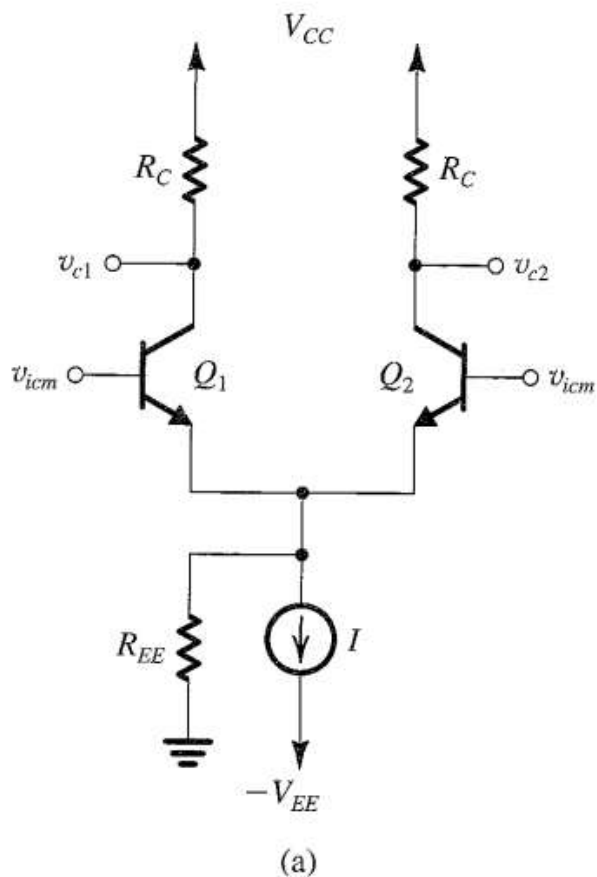


## Differential -Mode



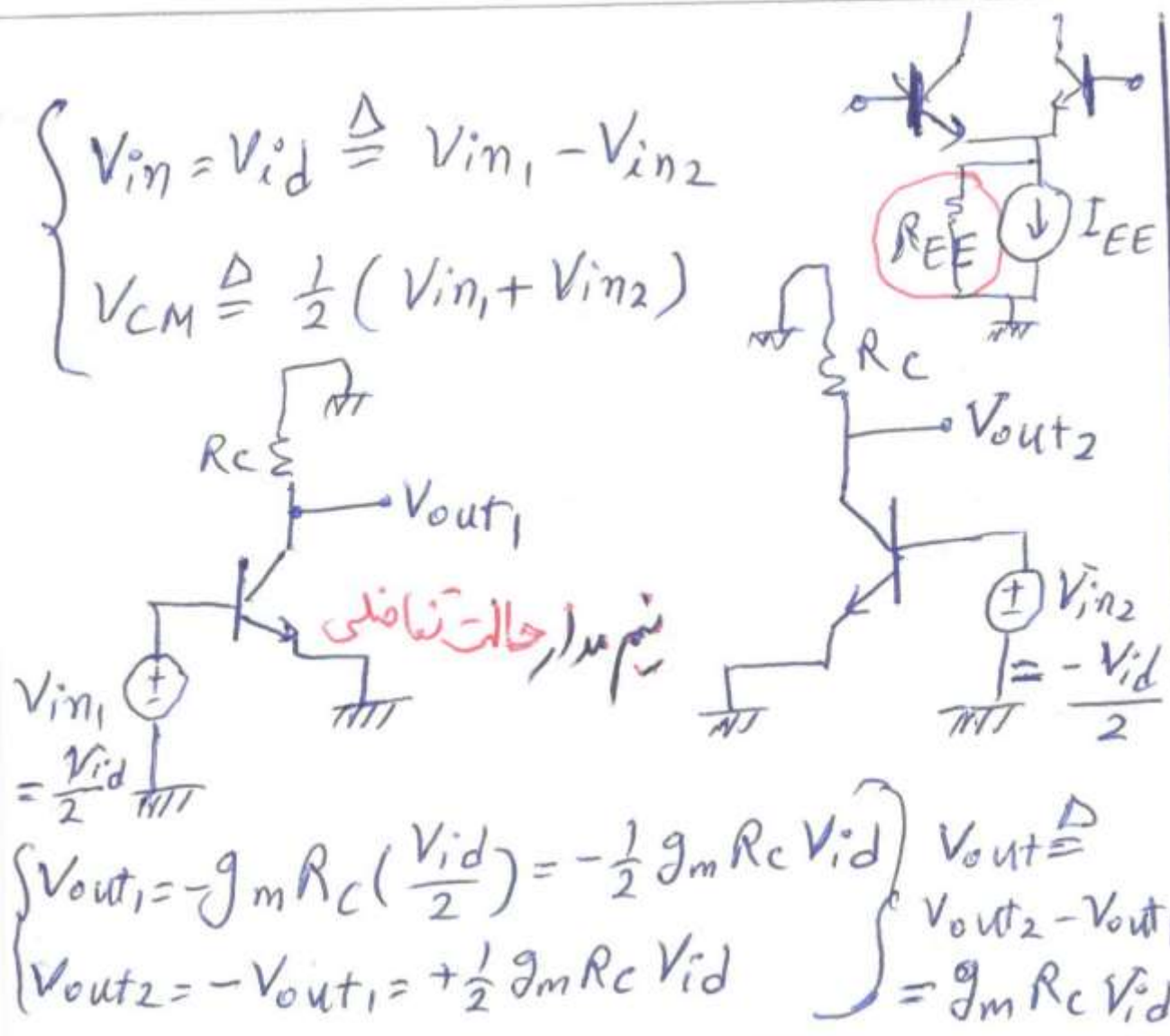


# Common-Mode





## یادآوری ۴: نیم مدار حالت تفاضلی

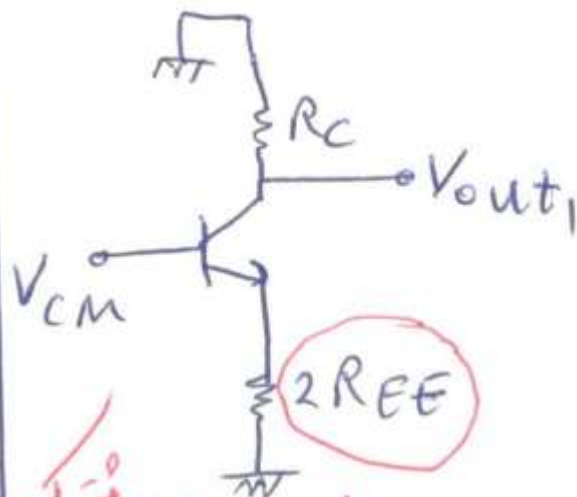
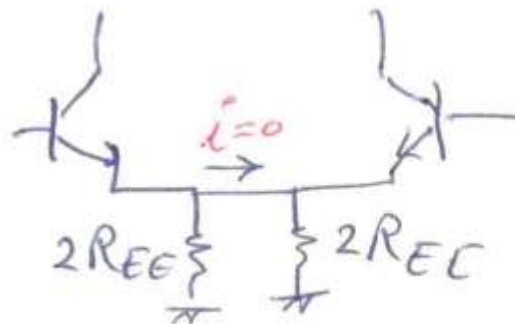




## یادآوری ۵: نیم مدار حالت مشترک

برای حالت مشترک

$$V_{in1} = V_{in2} = V_{CM}$$



$$V_{out1} = V_{out2} =$$
$$= \frac{-R_C}{\frac{1}{g_m} + 2R_{EE}} \cdot V_{CM}$$

نیم مدار حالت مشترک

$$V_{out} = V_{out2} - V_{out1} = 0$$

در حالت مشترک



شرط سکتال حرکت برای تقریب کشنده نااضلی BJT

$$V_{id} \ll 2V_T \Rightarrow \left( \frac{1}{2} V_{id} \right) \ll V_T$$

همرودۀ خطی برای BJT ها، ضلی حرکت است.

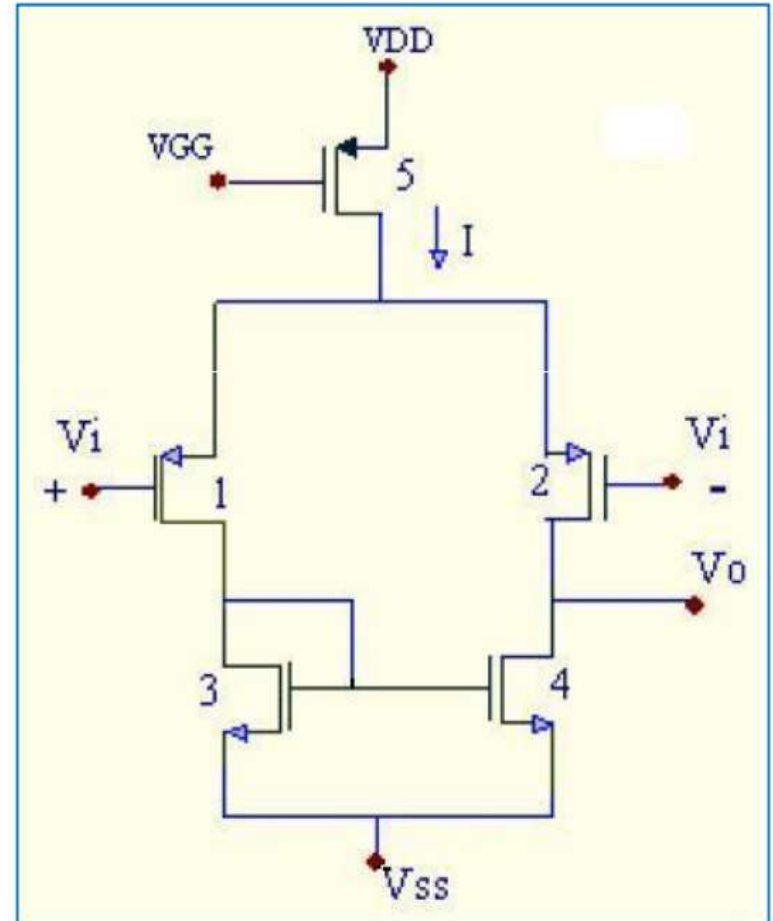
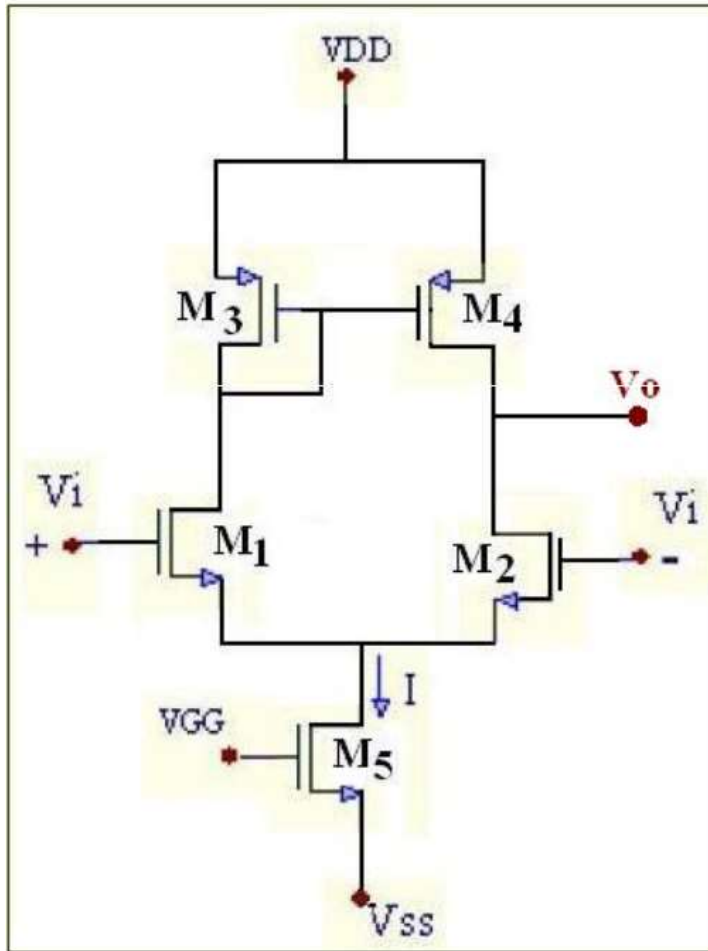
\* در اکثر مقدار ولتاژ حالت مشترک ورودی:

از روی مدار اشباع ترانزیستور بدست می آید.



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## MOS Differential Amplifier





# MOS Differential Amplifier

بدلیل کمتر بودن موبلیتی الکترونها در PMOS، از نظر **سرعت** کار ترانزیستور، **NMOS** برتری دارد.

مزیت استفاده از **PMOS** در ورودی: کمتر بودن سطح نویز این طبق نسبت به NMOS است.

**MOS** ها در زوج تفاضلی ورودی، از نظر **بزرگ بودن S.R.** نسبت به BJT ها یک مزیت است. از نظر **زیاد بودن ولتاژ آفست**، **برعکس** است.

$$K' = \mu \cdot C_{ox} \quad K'_n \approx 17, \quad K'_p \approx 8 \quad (\mu A/v^2)$$

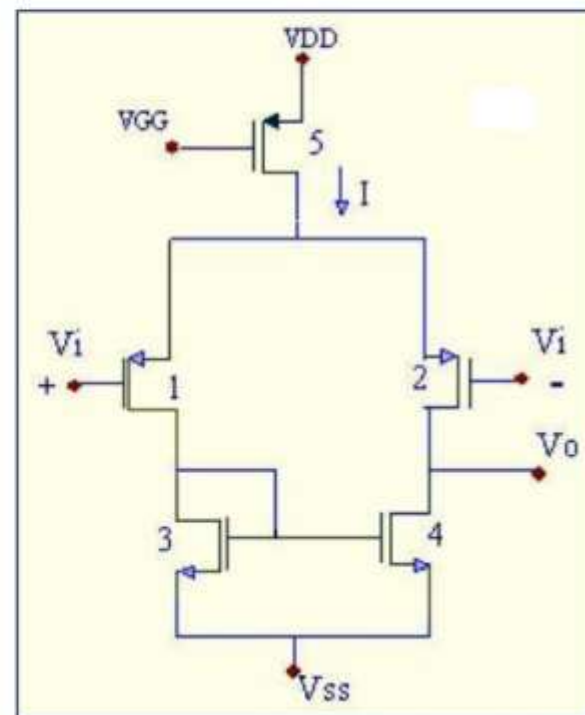
$$r_{out} = r_{ds2} \parallel r_{ds4}$$

$$A_v = \frac{V_o}{V_i} = g_{m1} \cdot r_{out} = g_{m1} (r_{ds2} \parallel r_{ds4})$$

$$CMR = V_i(\max) - V_i(\min)$$

$$V_i(\max) = V_{DD} - V_{SD5}(\text{sat}) - |V_{T1}| - \sqrt{\frac{2I_{D1}}{\beta_1}}$$

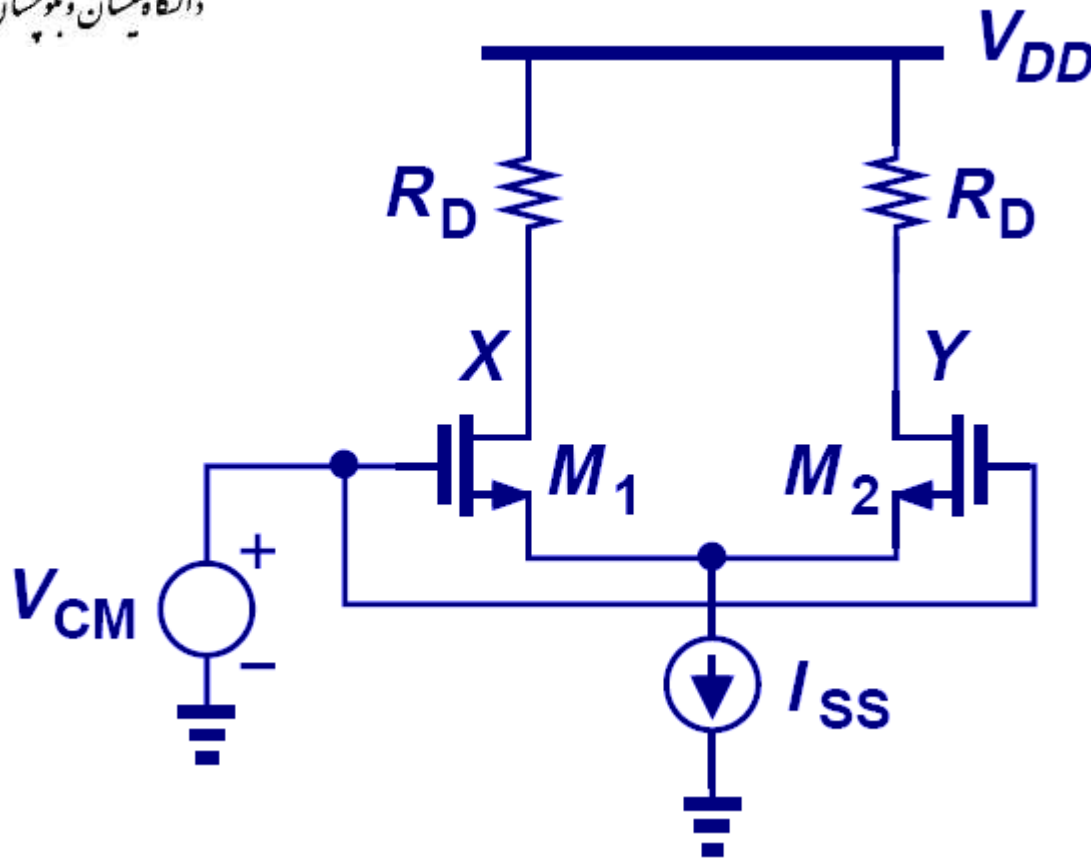
$$V_i(\min) = V_{SS} + V_{T3} + \sqrt{\frac{2I_{D3}}{\beta_3}} - |V_{T1}|$$





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## MOS Differential Pair's Common-Mode Response



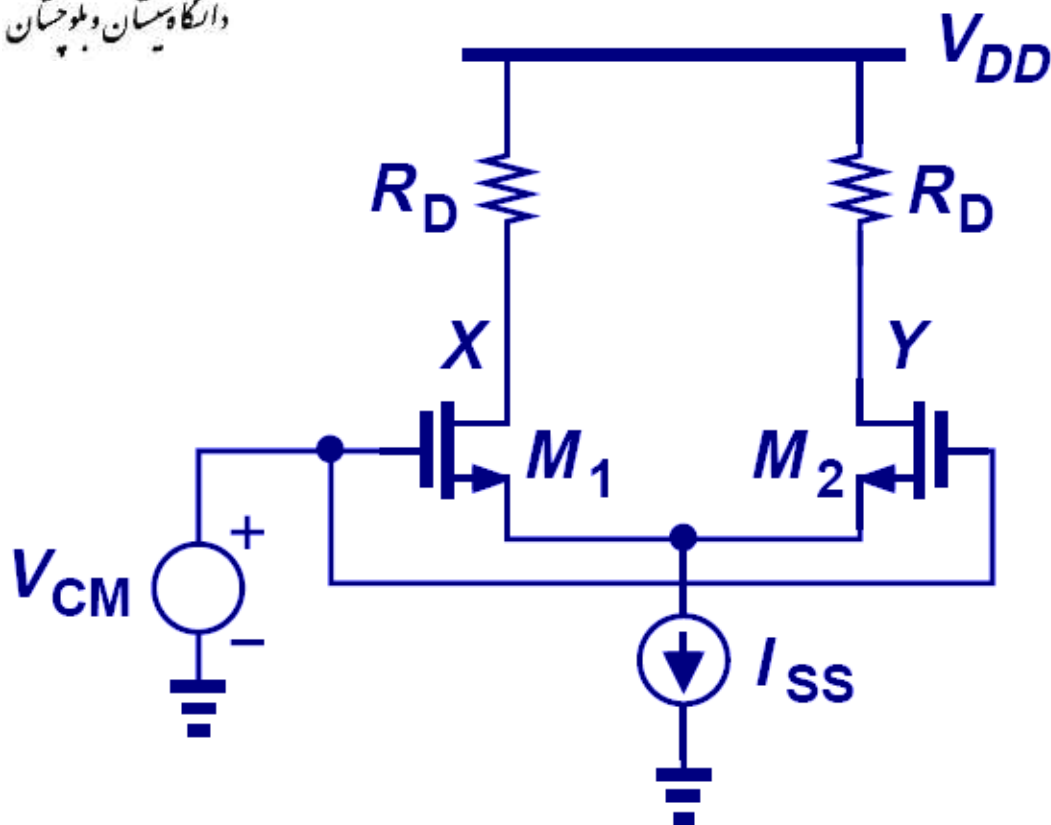
$$V_X = V_Y = V_{DD} - R_D \frac{I_{SS}}{2}$$

- Similar to its bipolar counterpart, MOS differential pair produces zero differential output as  $V_{CM}$  changes.



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## Equilibrium Overdrive Voltage



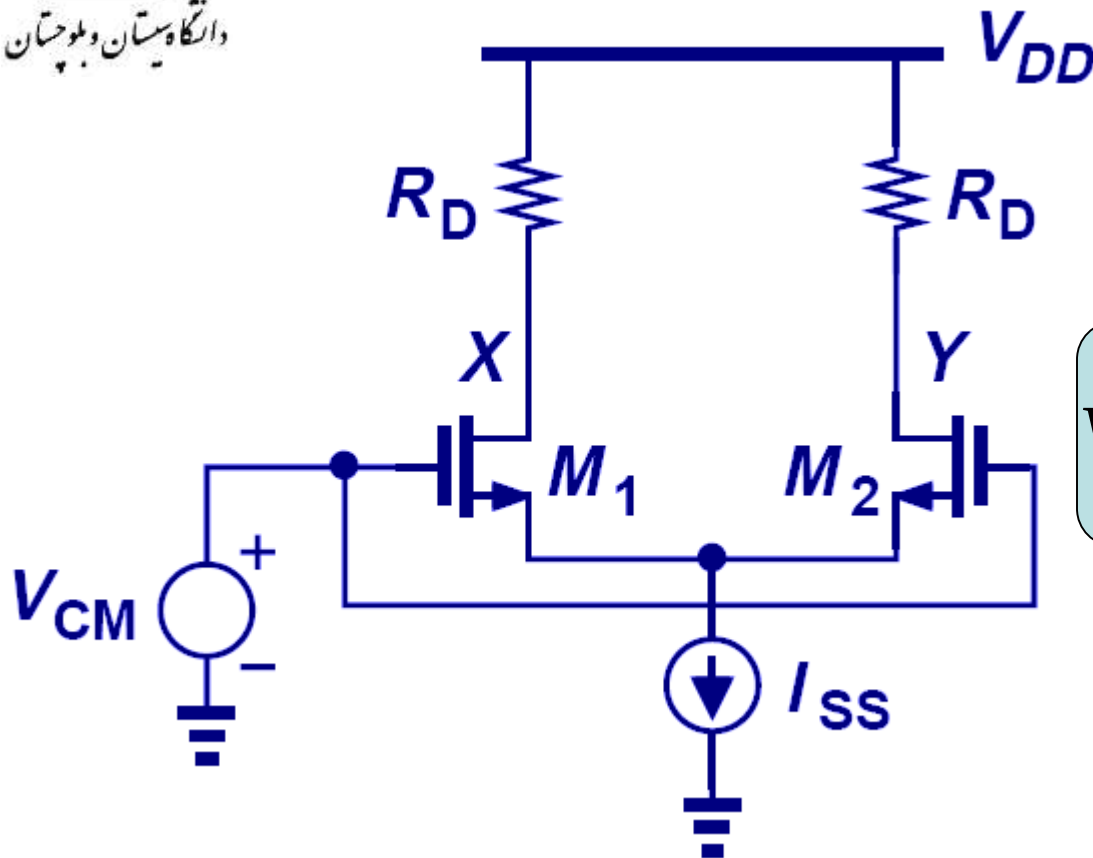
$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

- The equilibrium overdrive voltage is defined as the overdrive voltage seen by  $M_1$  and  $M_2$  when both of them carry a current of  $I_{SS}/2$ .



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## Minimum Common-mode Output Voltage



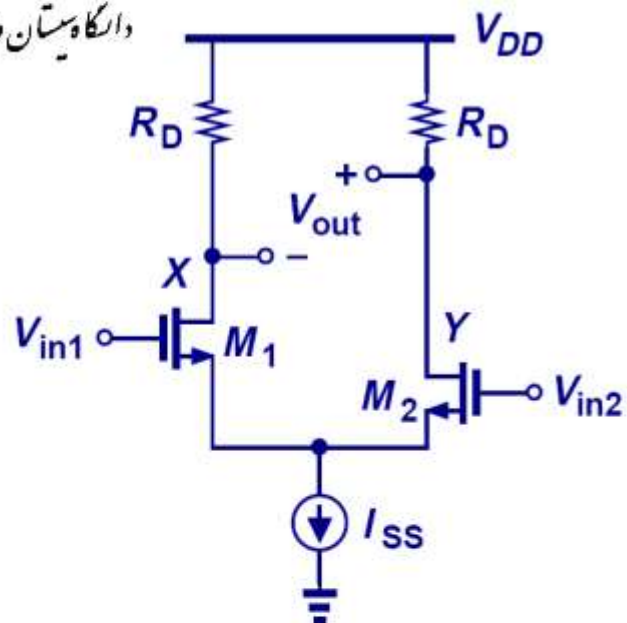
$$V_{DD} - R_D \frac{I_{SS}}{2} > V_{CM} - V_{TH}$$

- In order to maintain  $M_1$  and  $M_2$  in saturation, the common-mode output voltage cannot fall below the value above.
- This value usually limits voltage gain.

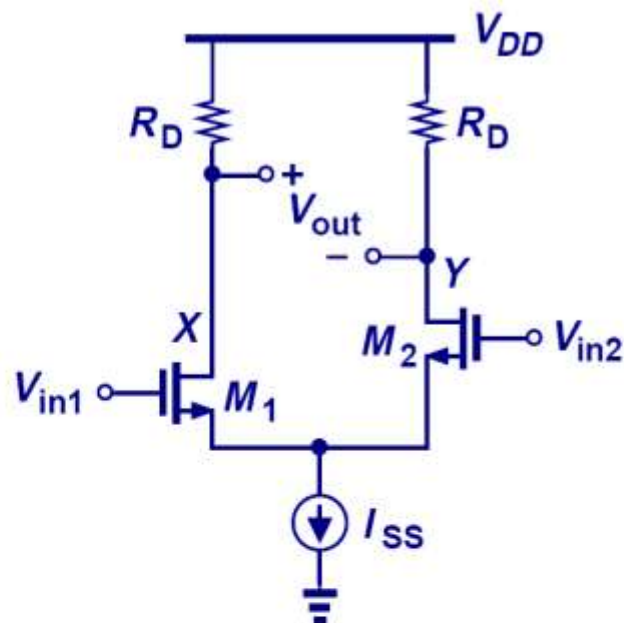


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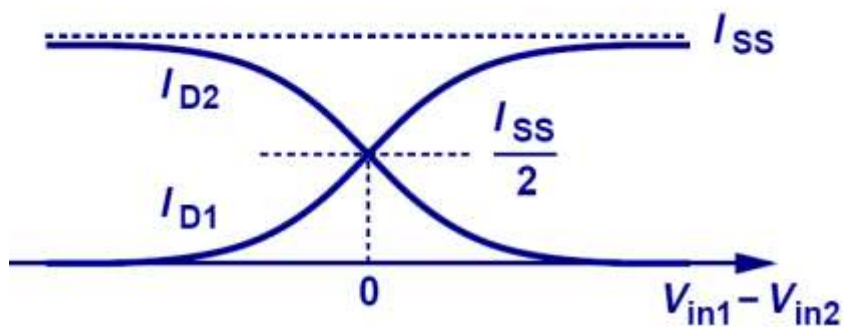
# Differential Response



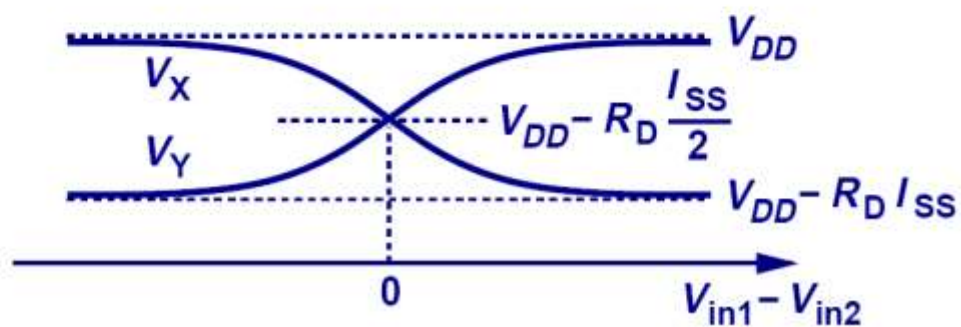
(a)



(b)



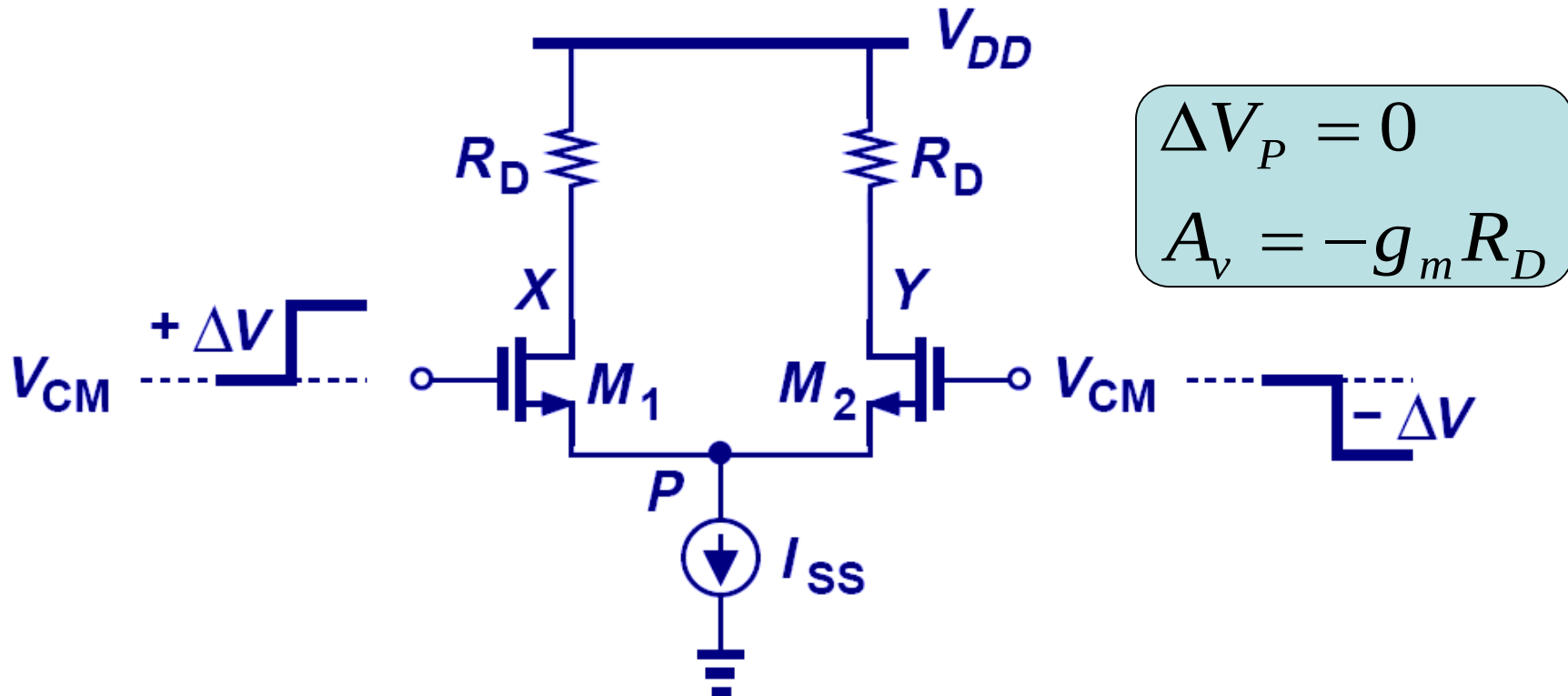
(c)





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## Small-Signal Response

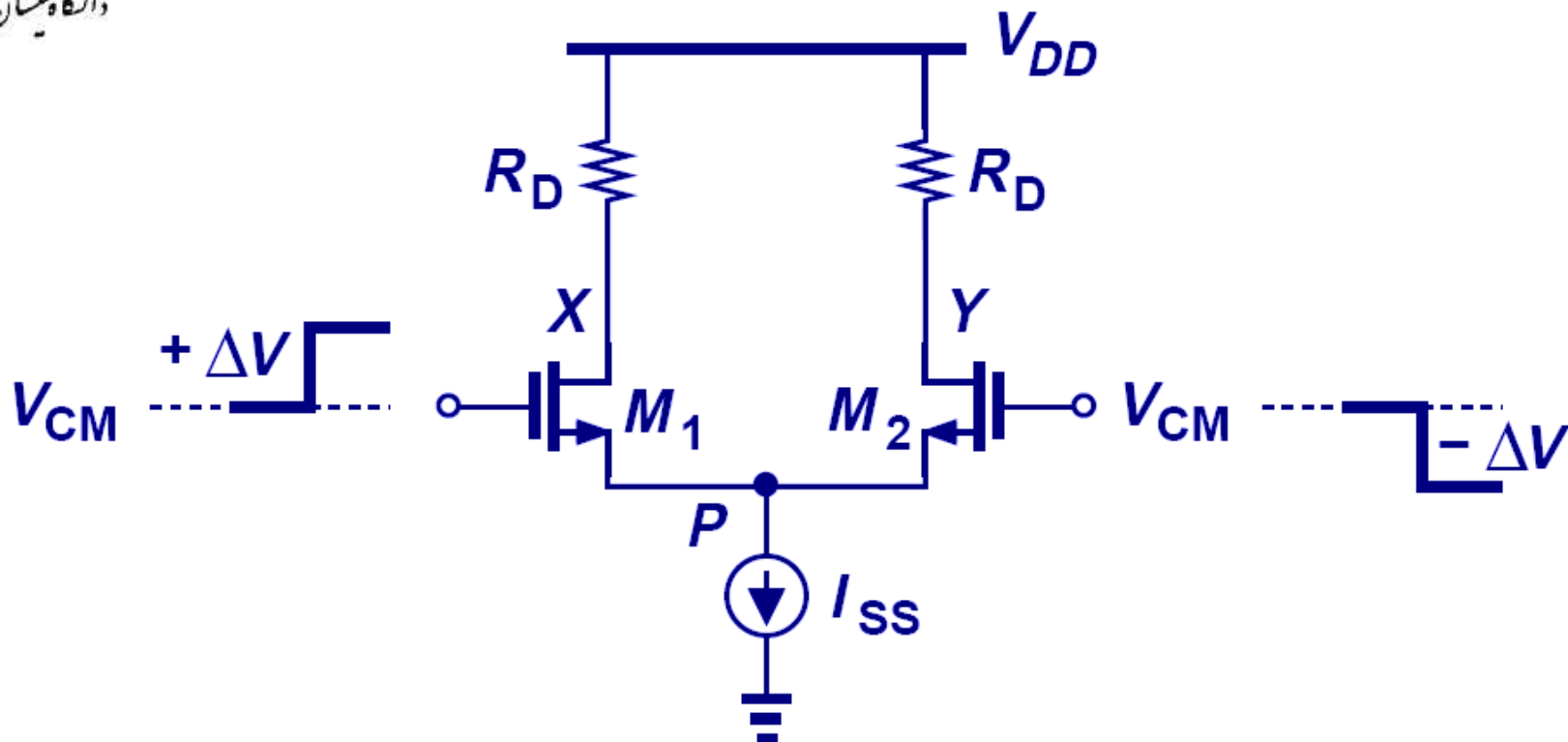


- Similar to its bipolar counterpart, the MOS differential pair exhibits the same virtual ground node and small signal gain.



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## Power and Gain Tradeoff

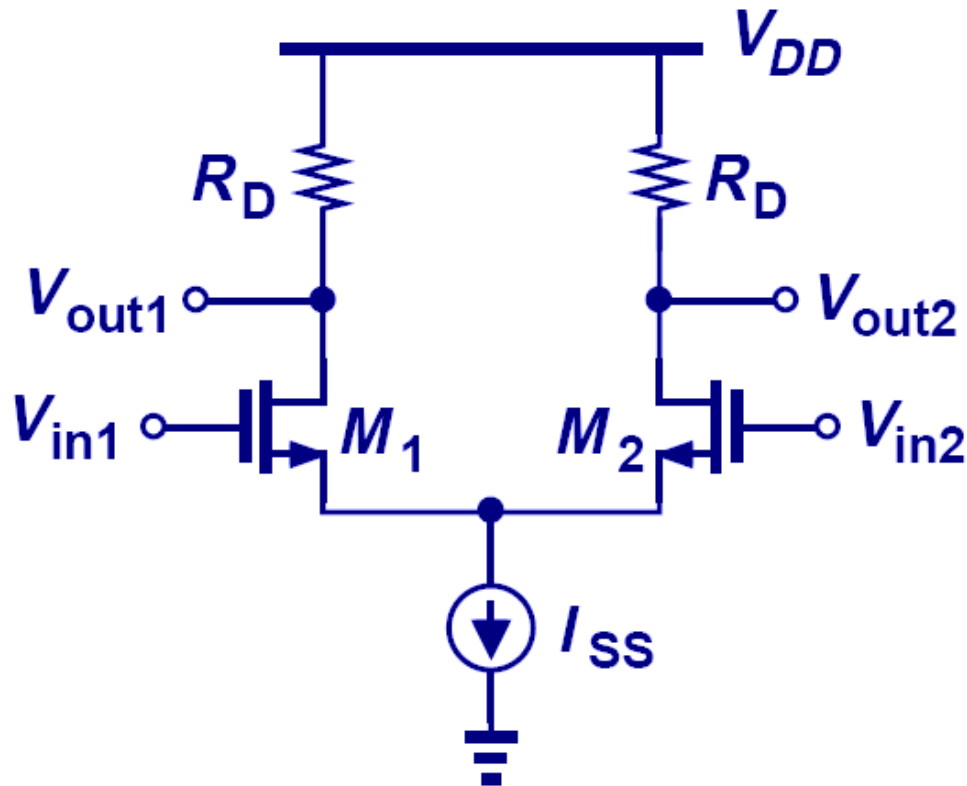


- In order to obtain the source gain as a CS stage, a MOS differential pair must dissipate twice the amount of current. This power and gain tradeoff is also echoed in its bipolar counterpart.



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# MOS Differential Pair's Large-Signal Response

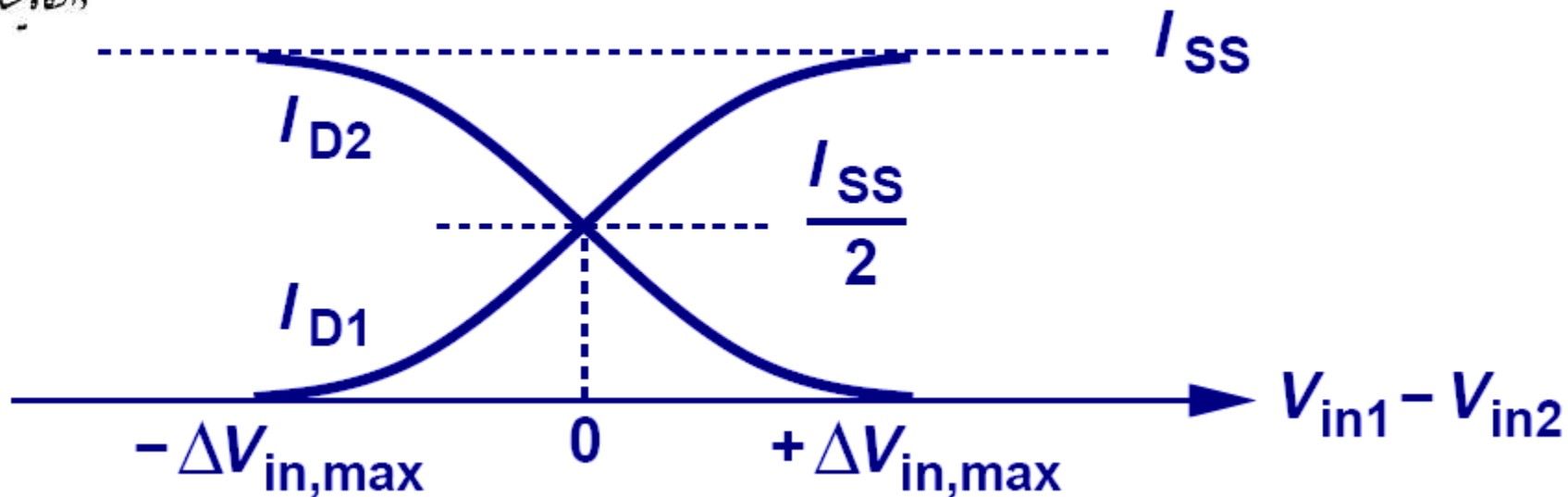


$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$



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## Maximum Differential Input Voltage



$$|V_{in1} - V_{in2}|_{\max} = \sqrt{2}(V_{GS} - V_{TH})_{\text{equil}}$$

- There exists a finite differential input voltage that completely steers the tail current from one transistor to the other. This value is known as the maximum differential input voltage.

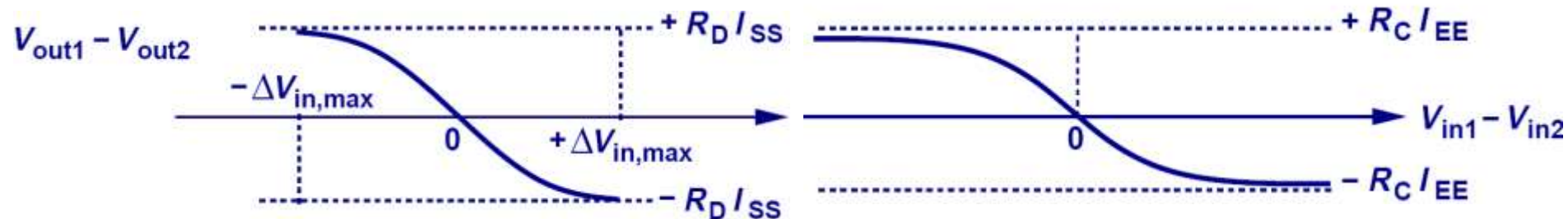


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## Contrast Between MOS and Bipolar Differential Pairs

*MOS*

*Bipolar*

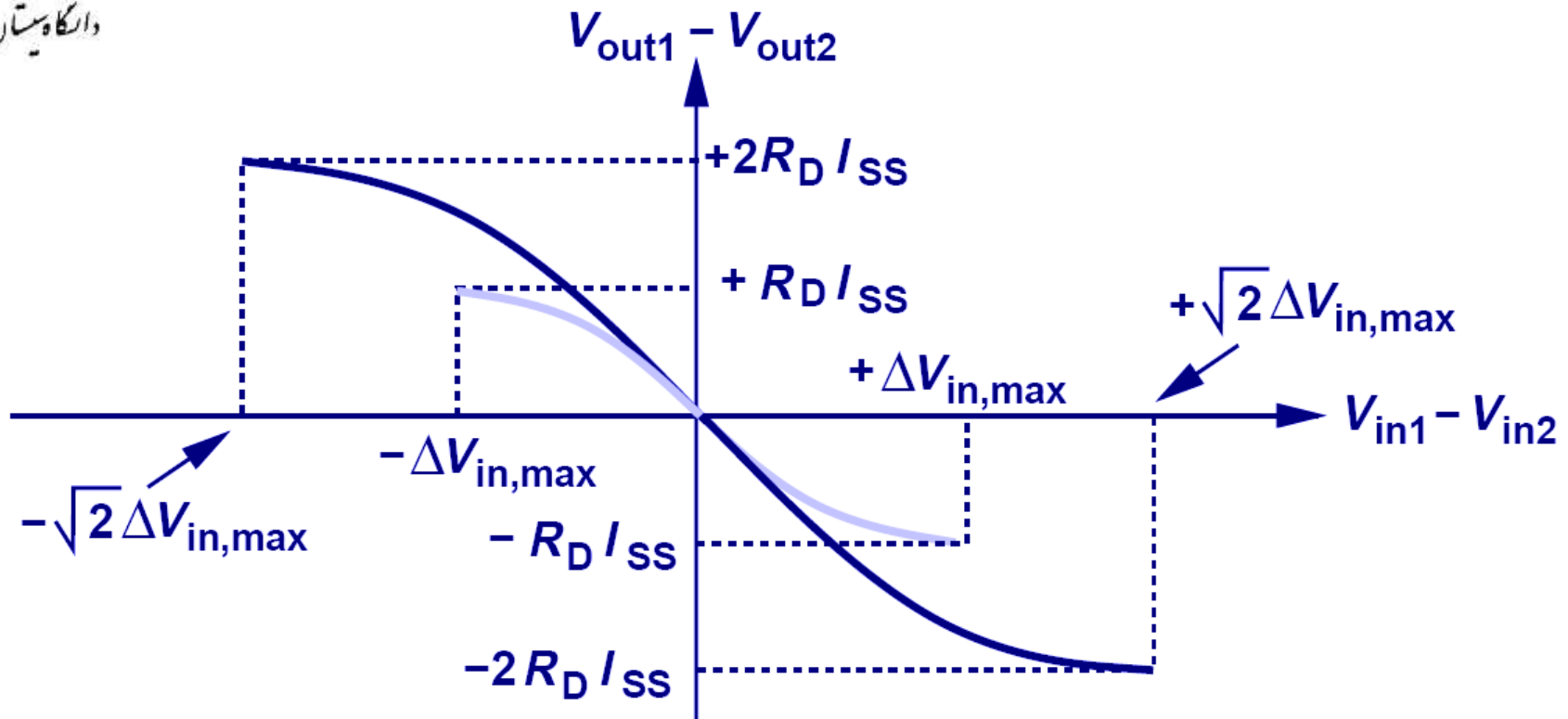


- In a MOS differential pair, there exists a finite differential input voltage to completely switch the current from one transistor to the other, whereas, in a bipolar pair that voltage is infinite.



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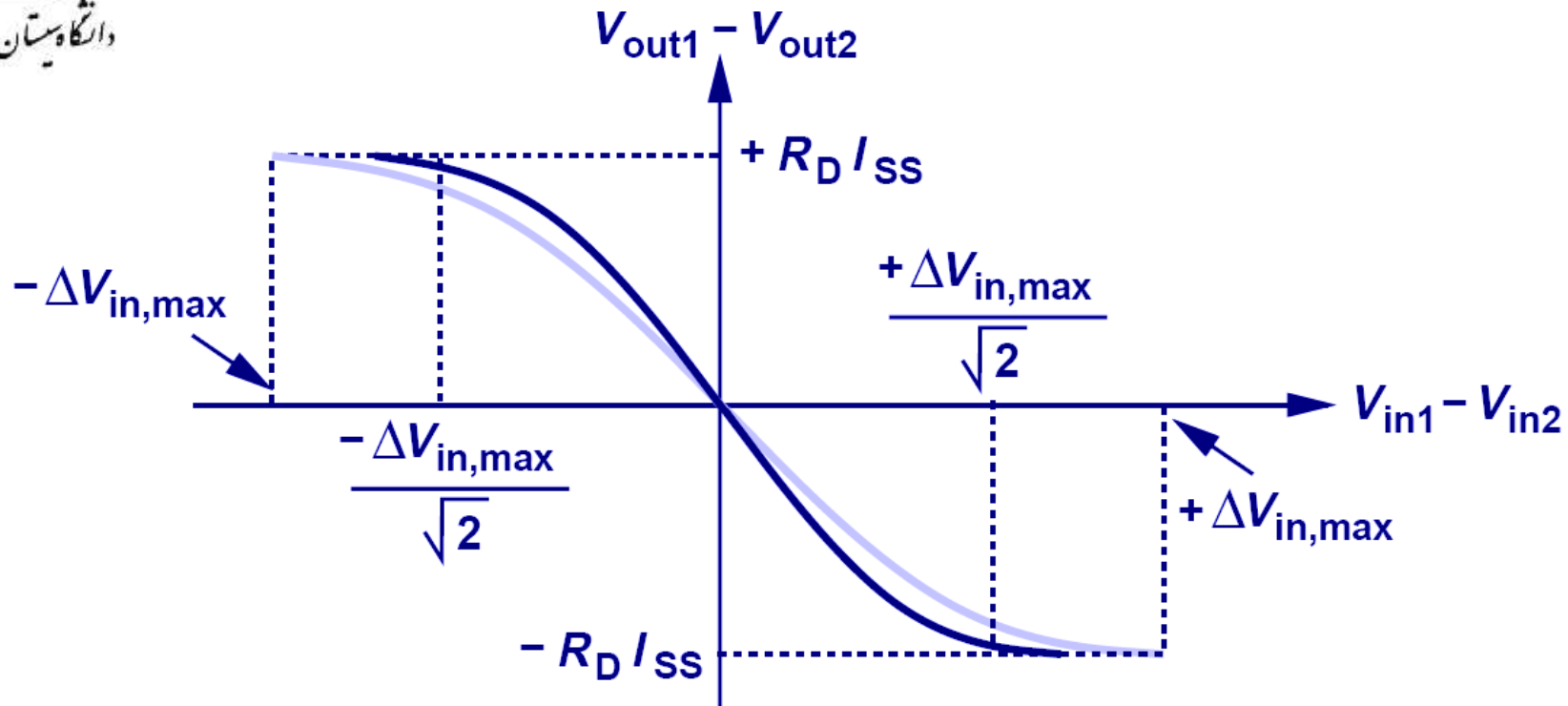
## The effects of Doubling the Tail Current



- Since  $I_{SS}$  is doubled and  $W/L$  is unchanged, the equilibrium overdrive voltage for each transistor must increase by  $\sqrt{2}$  to accommodate this change, thus  $\Delta V_{in,max}$  increases by  $\sqrt{2}$  as well. Moreover, since  $I_{SS}$  is doubled, the differential output swing will double.



## The effects of Doubling W/L

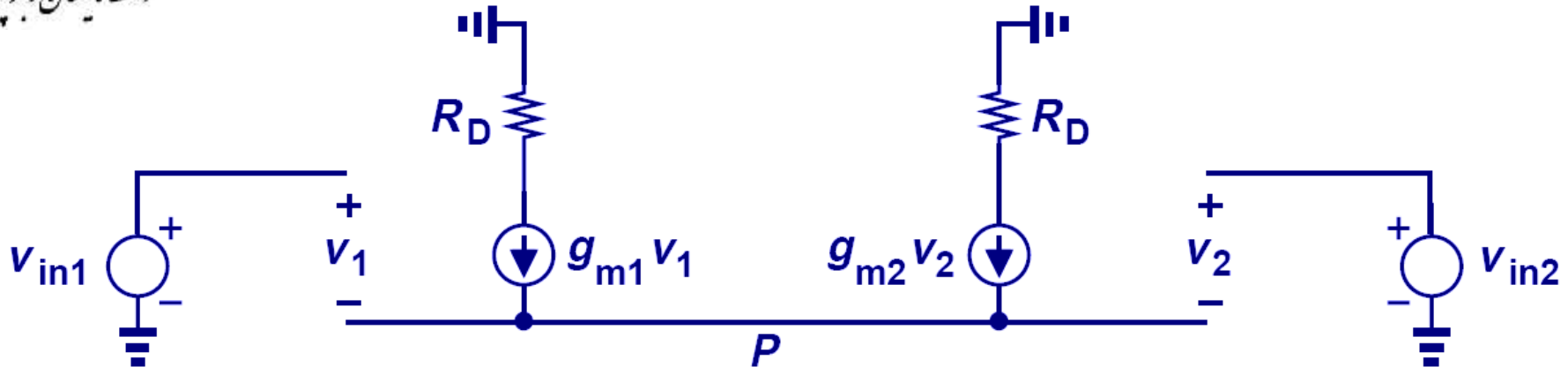


- Since  $W/L$  is doubled and the tail current remains unchanged, the equilibrium overdrive voltage will be lowered by  $\sqrt{2}$  to accommodate this change, thus  $\Delta V_{in,max}$  will be lowered by  $\sqrt{2}$  as well. Moreover, the differential output swing will remain unchanged since neither  $I_{SS}$  nor  $R_D$  has changed



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## Small-Signal Analysis of MOS Differential Pair



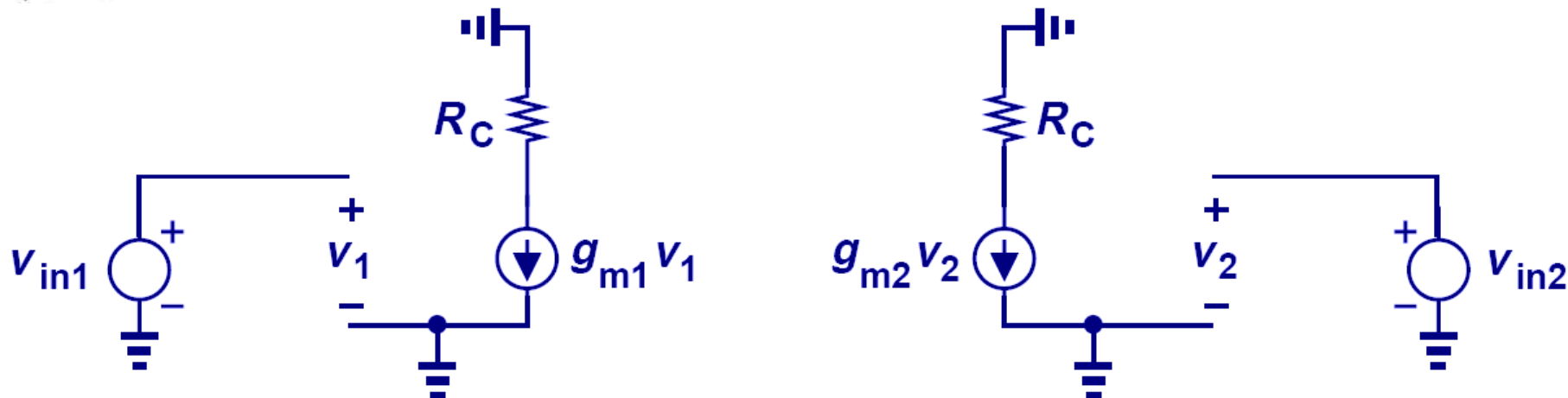
$$I_{D1} - I_{D2} \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} (V_{in1} - V_{in2})$$

- When the input differential signal is small compared to  $4I_{SS}/\mu_n C_{ox}(W/L)$ , the output differential current is linearly proportional to it, and small-signal model can be applied.



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## Virtual Ground and Half Circuit

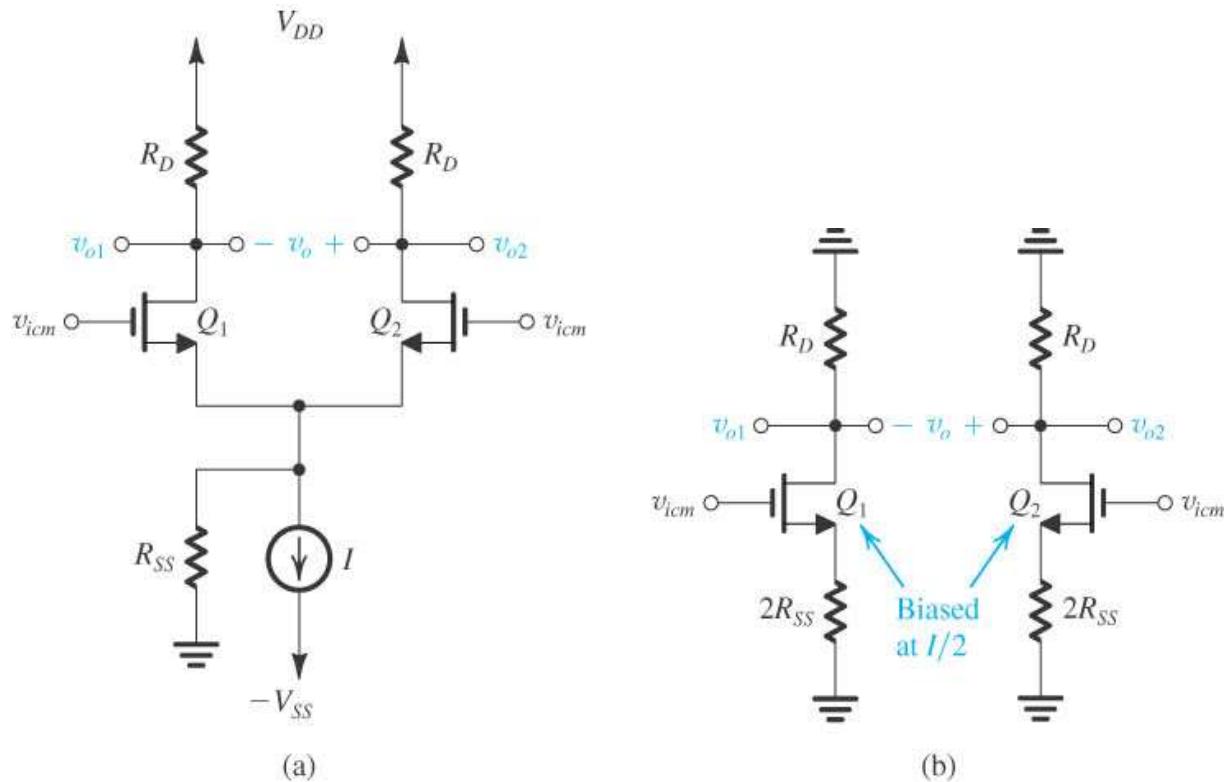


$$\Delta V_P = 0$$

$$A_v = -g_m R_C$$

- Applying the same analysis as the bipolar case, we will arrive at the same conclusion that node P will not move for small input signals and the concept of half circuit can be used to calculate the gain.

# Common-mode input signal



(a) The MOS differential amplifier with a common-mode input signal  $v_{icm}$

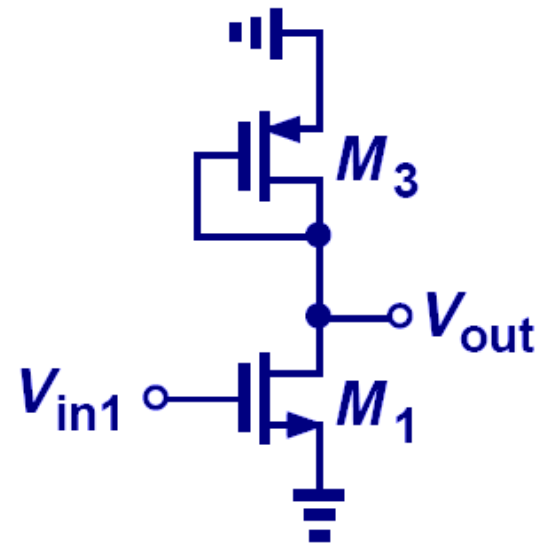
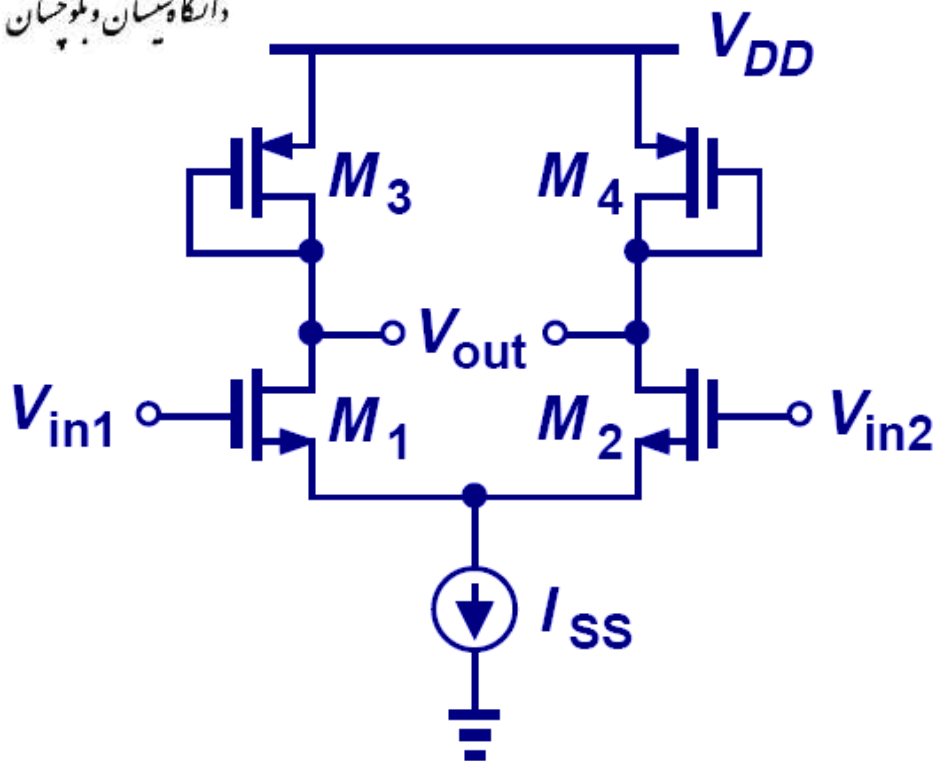
(b) Equivalent circuit for determining the common-mode gain (with  $r_o$  ignored).

Each half of the circuit is known as the “**common-mode half-circuit**.”



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## MOS Differential Pair Half Circuit Example I



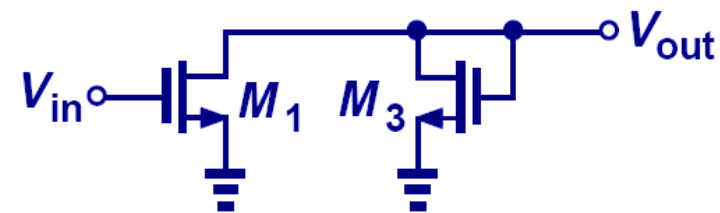
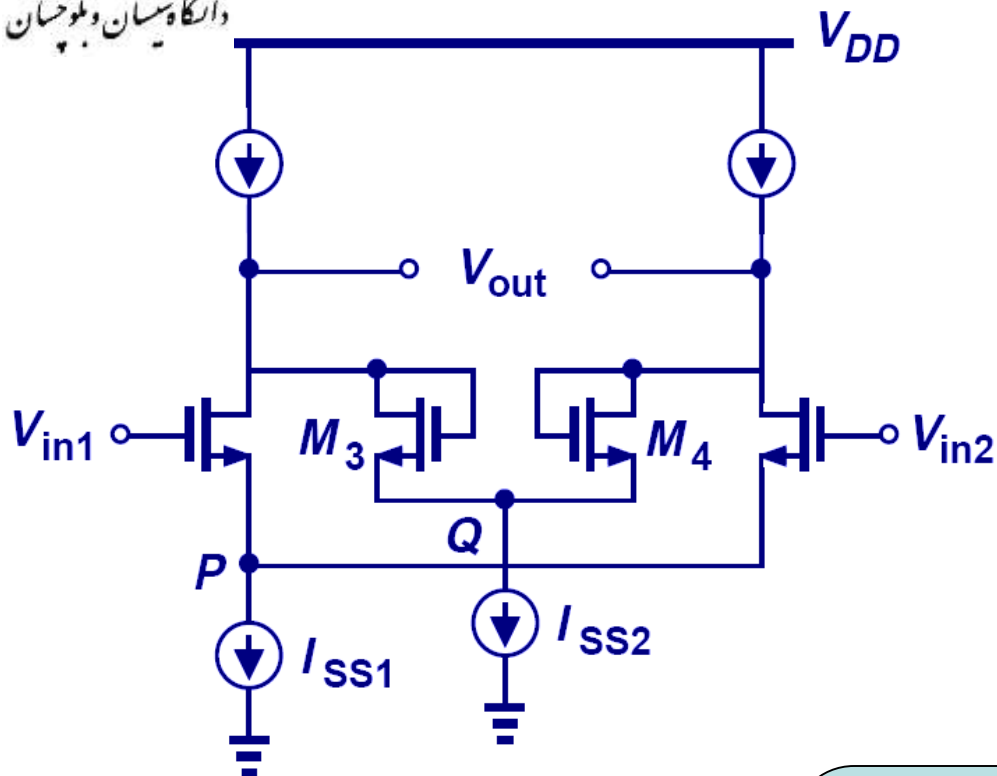
$$\lambda \neq 0$$

$$A_v = -g_{m1} \left( \frac{1}{g_{m3}} \parallel r_{O3} \parallel r_{O1} \right)$$



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# MOS Differential Pair Half Circuit Example II



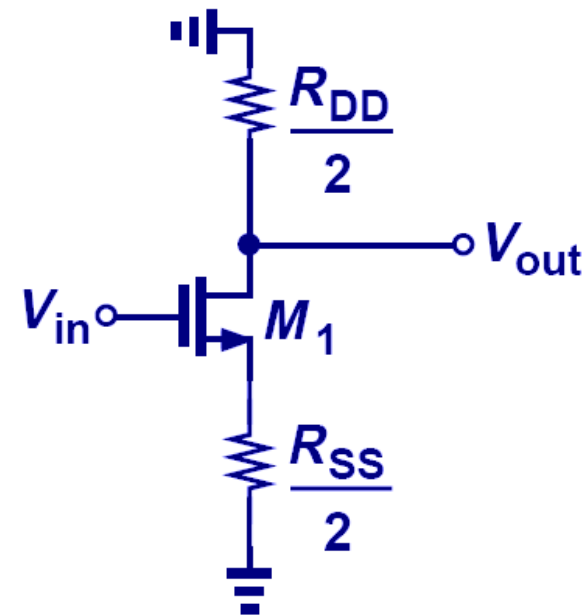
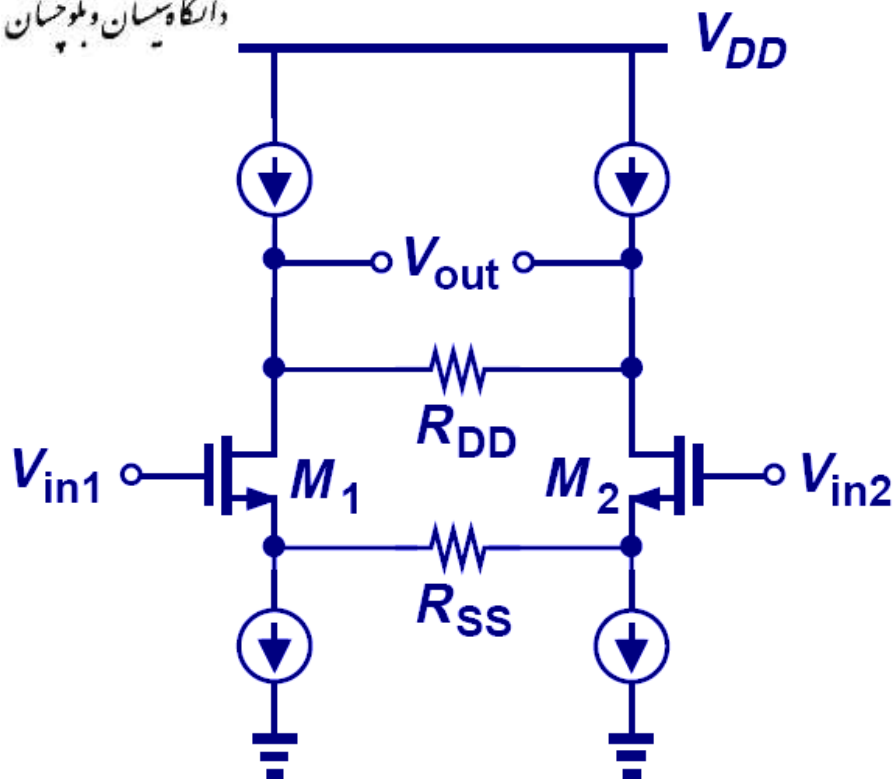
$$\lambda = 0$$

$$A_v = -\frac{g_{m1}}{g_{m3}}$$



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## MOS Differential Pair Half Circuit Example III



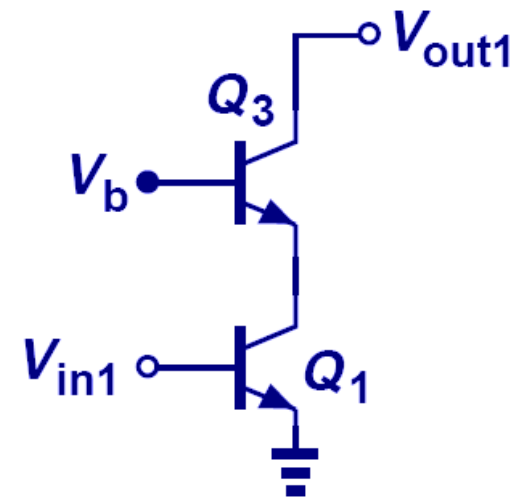
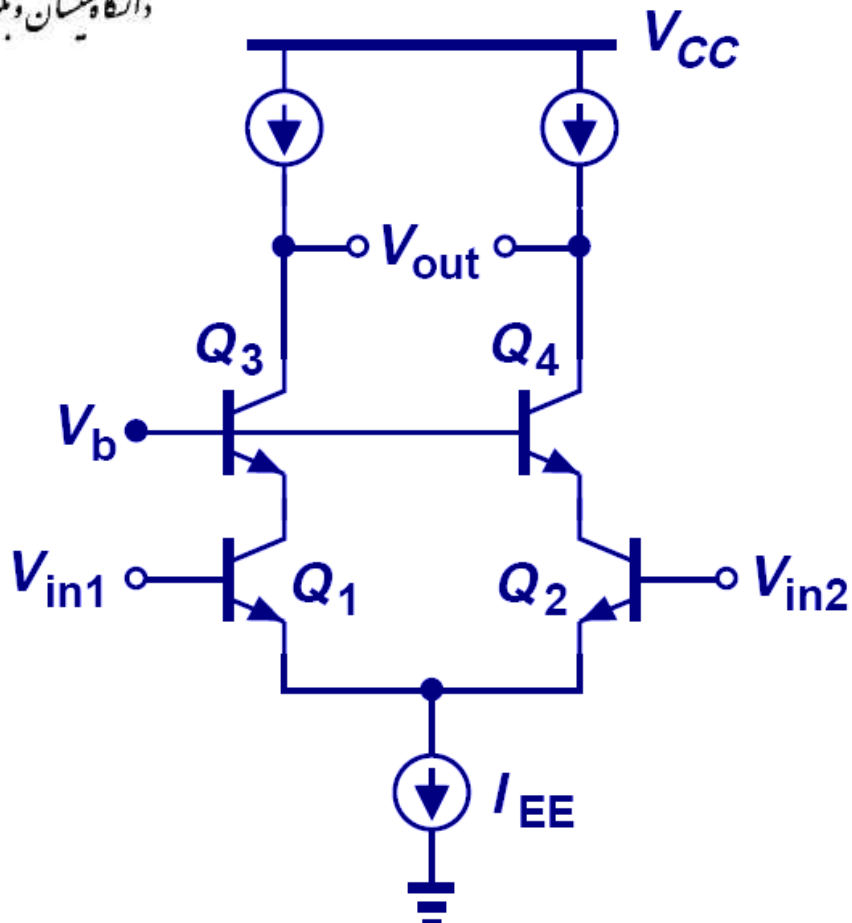
$$\lambda = 0$$

$$A_v = -\frac{R_{DD}/2}{R_{SS}/2 + 1/g_m}$$



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# Bipolar Cascode Differential Pair

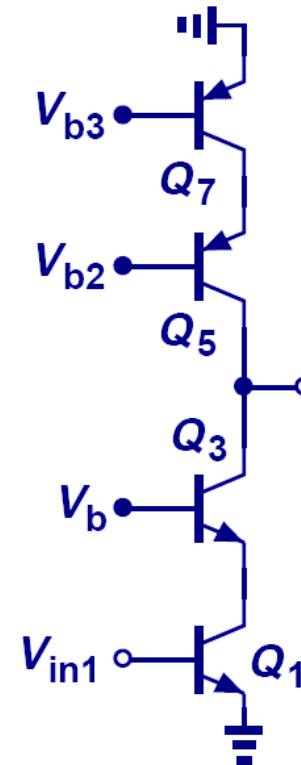
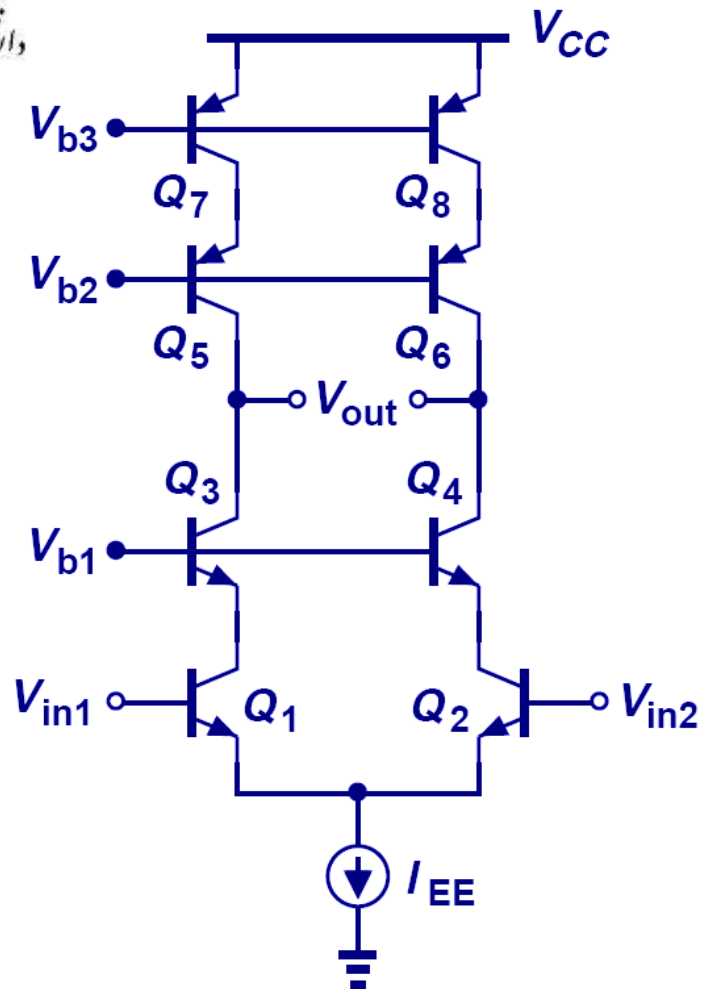


$$A_v = -g_{m1} [g_{m3} (r_{O1} \parallel r_{\pi3}) r_{O3} + r_{O1}]$$



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## Bipolar Telescopic Cascode

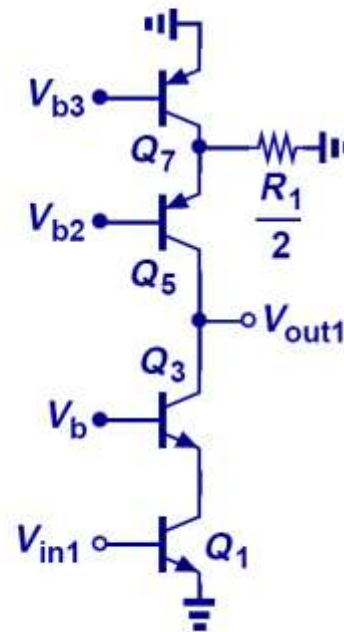
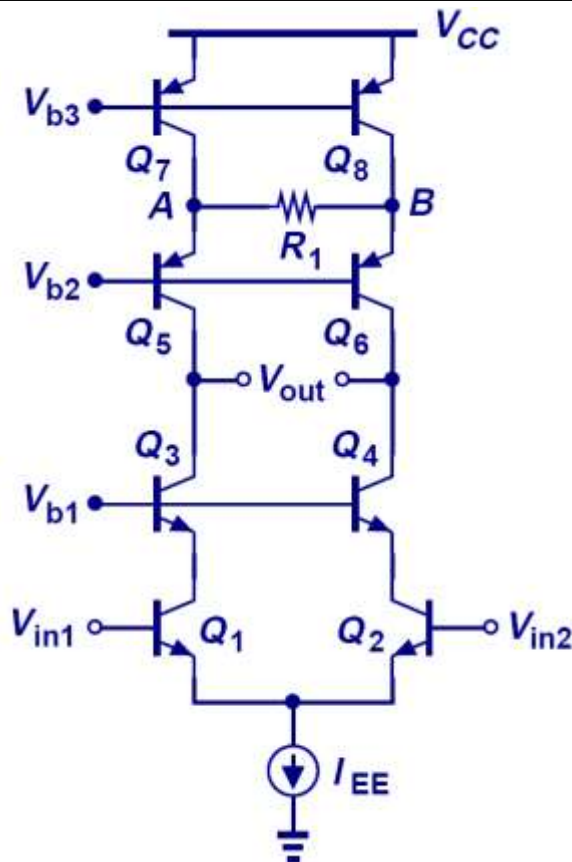


$$A_v \approx -g_{m1} [g_{m3} r_{O3} (r_{O1} \parallel r_{\pi3})] \parallel [g_{m5} r_{O5} (r_{O7} \parallel r_{\pi5})]$$



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## Example: Bipolar Telescopic Parasitic Resistance



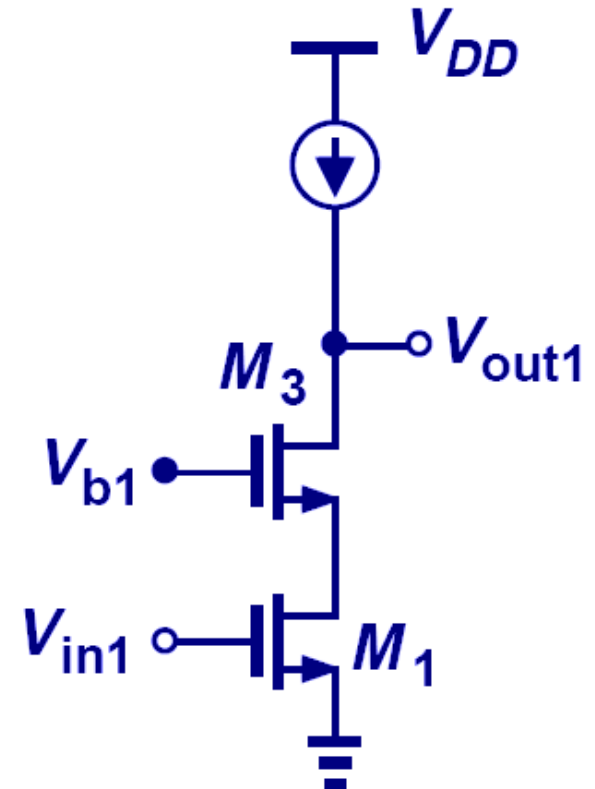
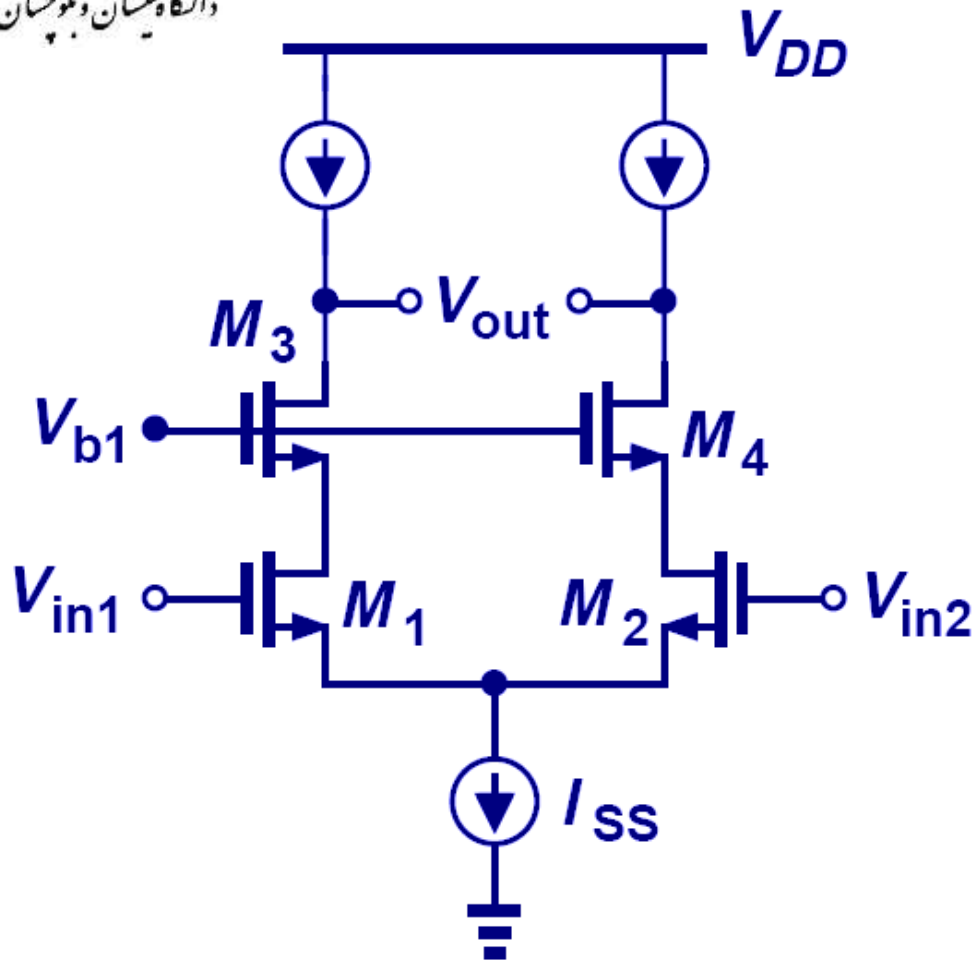
$$R_{op} = r_{O5} \left[ 1 + g_{m5} \left( r_{O7} \parallel r_{\pi5} \parallel \frac{R_1}{2} \right) \right] + r_{O7} \parallel r_{\pi5} \parallel \frac{R_1}{2}$$

$$A_v = -g_{m1} [g_{m3} r_{O3} (r_{O1} \parallel r_{\pi3})] \parallel R_{op}$$



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## MOS Cascode Differential Pair

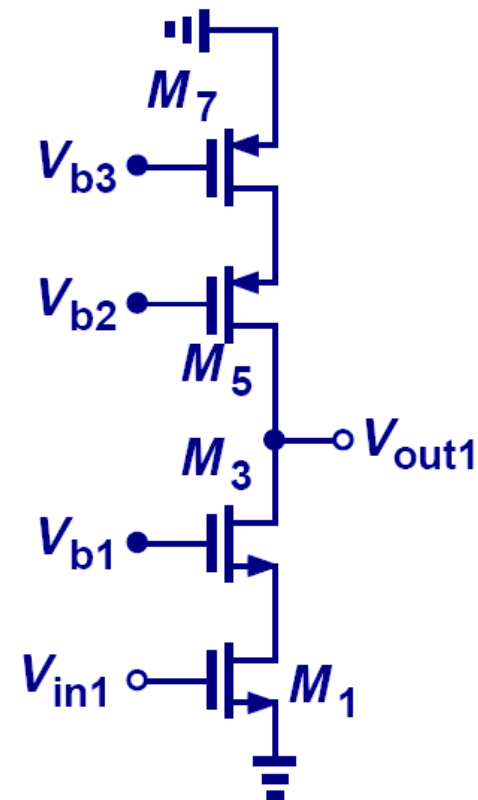
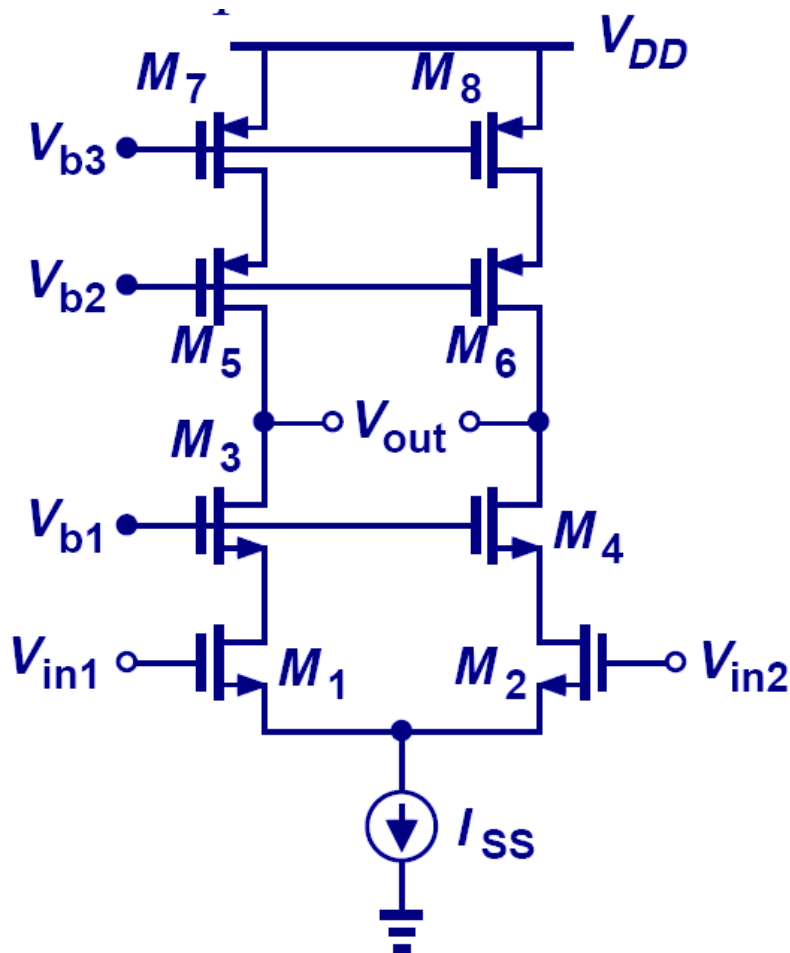


$$A_v \approx -g_{m1} r_{O3} g_{m3} r_{O1}$$



دانشگاه پشاور

# MOS Telescopic Cascode

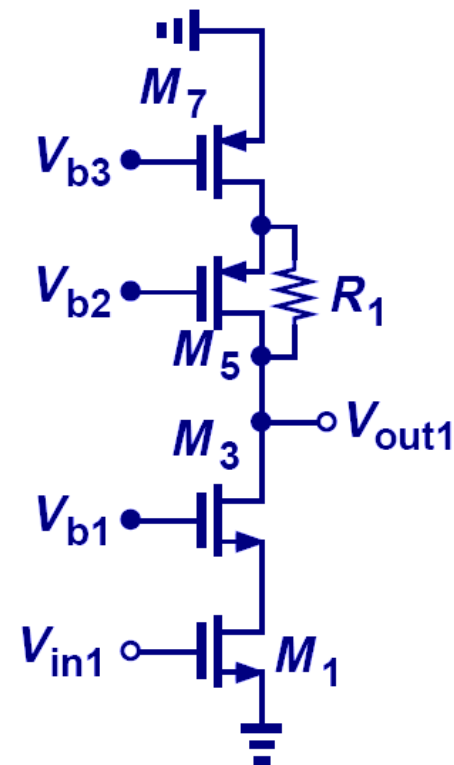
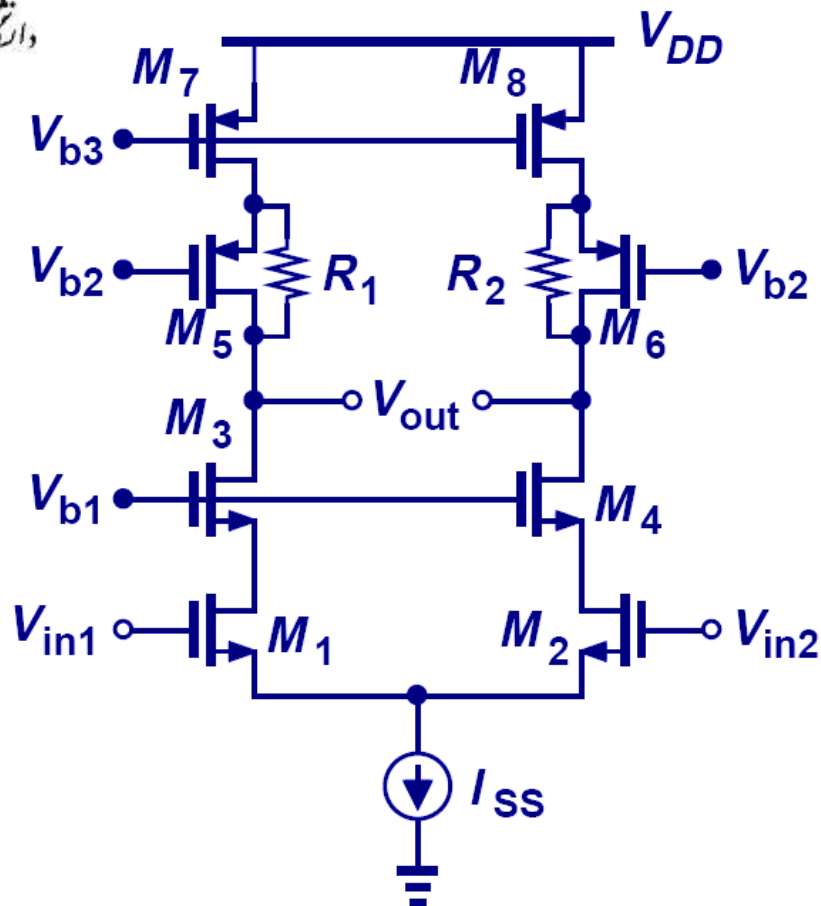


$$A_v \approx -g_{m1} [(g_{m3} r_{O3} r_{O1}) \parallel (g_{m5} r_{O5} r_{O7})]$$



دانشگاه مهاباد

## Example: MOS Telescopic Parasitic Resistance



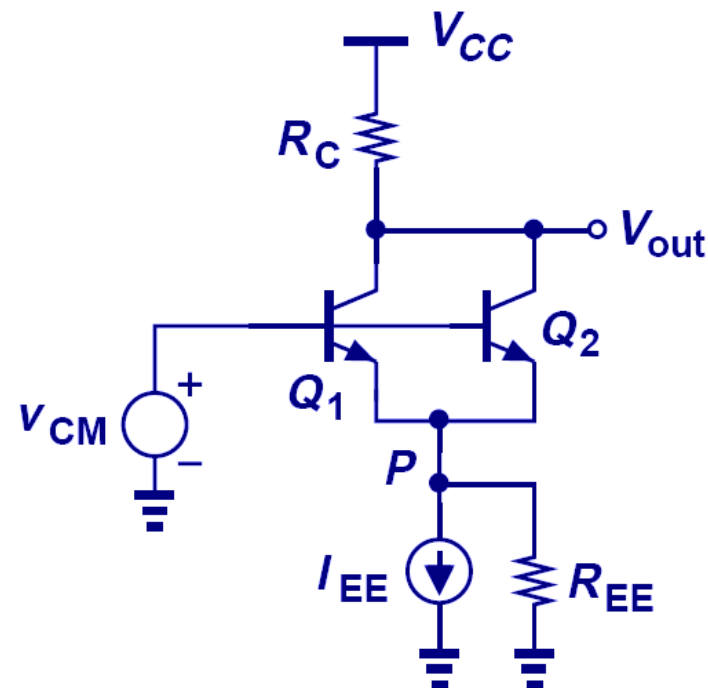
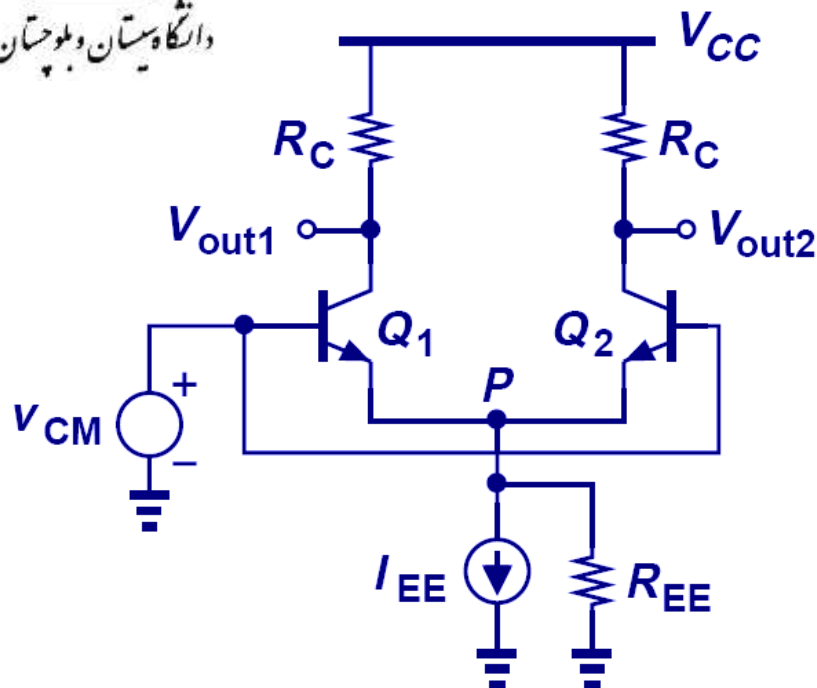
$$R_{op} = r_{O5} \parallel [R_1(1 + g_{m5}r_{O7}) + r_{O7}]$$

$$A_v \approx -g_{m1}(R_{op} \parallel r_{O3}g_{m3}r_{O1})$$



دانشگاه گجرات

## Effect of Finite Tail Impedance



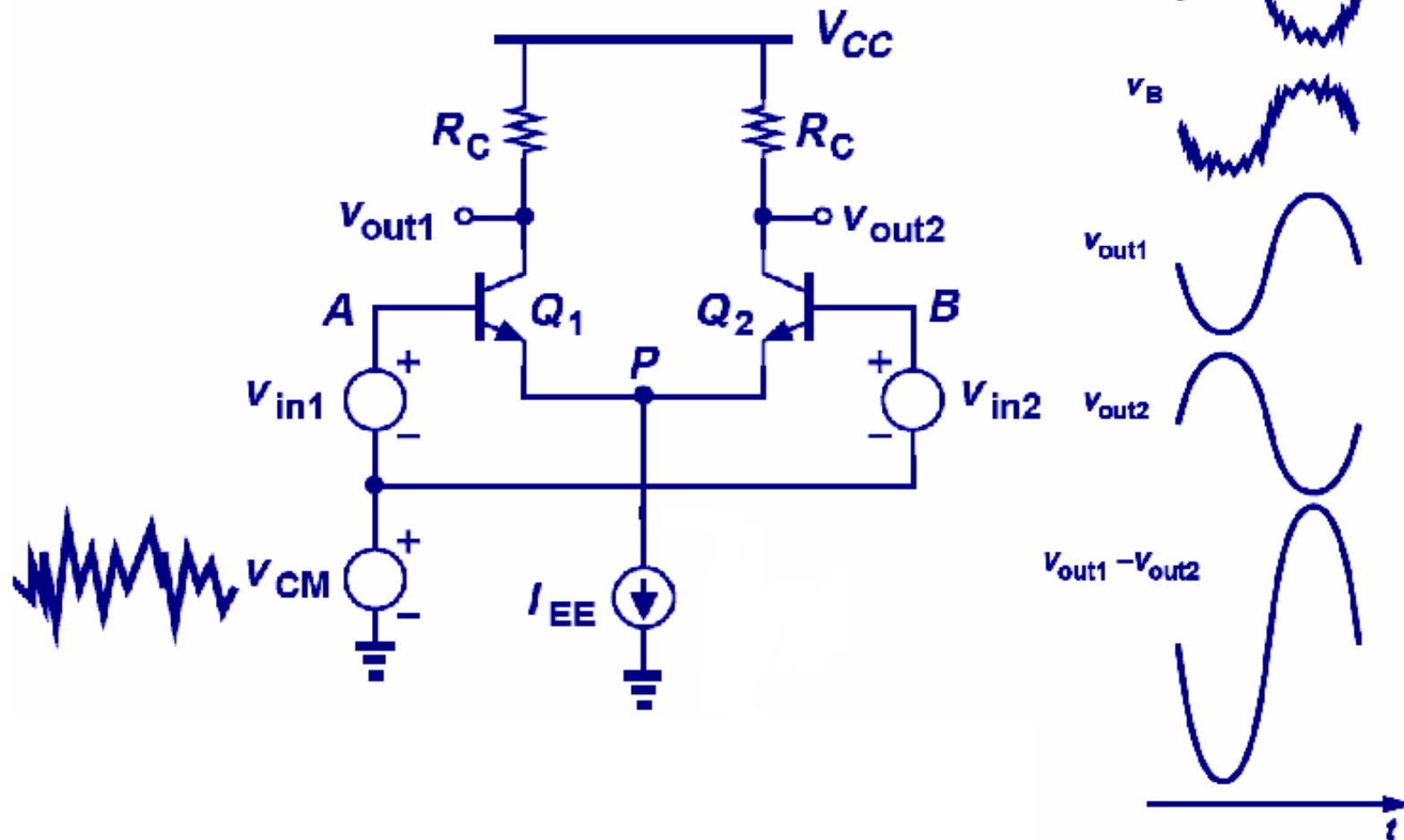
$$\frac{\Delta V_{out,CM}}{\Delta V_{in,CM}} = - \frac{R_C / 2}{R_{EE} + 1/2g_m}$$

- If the tail current source is not ideal, then when a input CM voltage is applied, the currents in  $Q_1$  and  $Q_2$  and hence output CM voltage will change.



دانشگاه پشاور

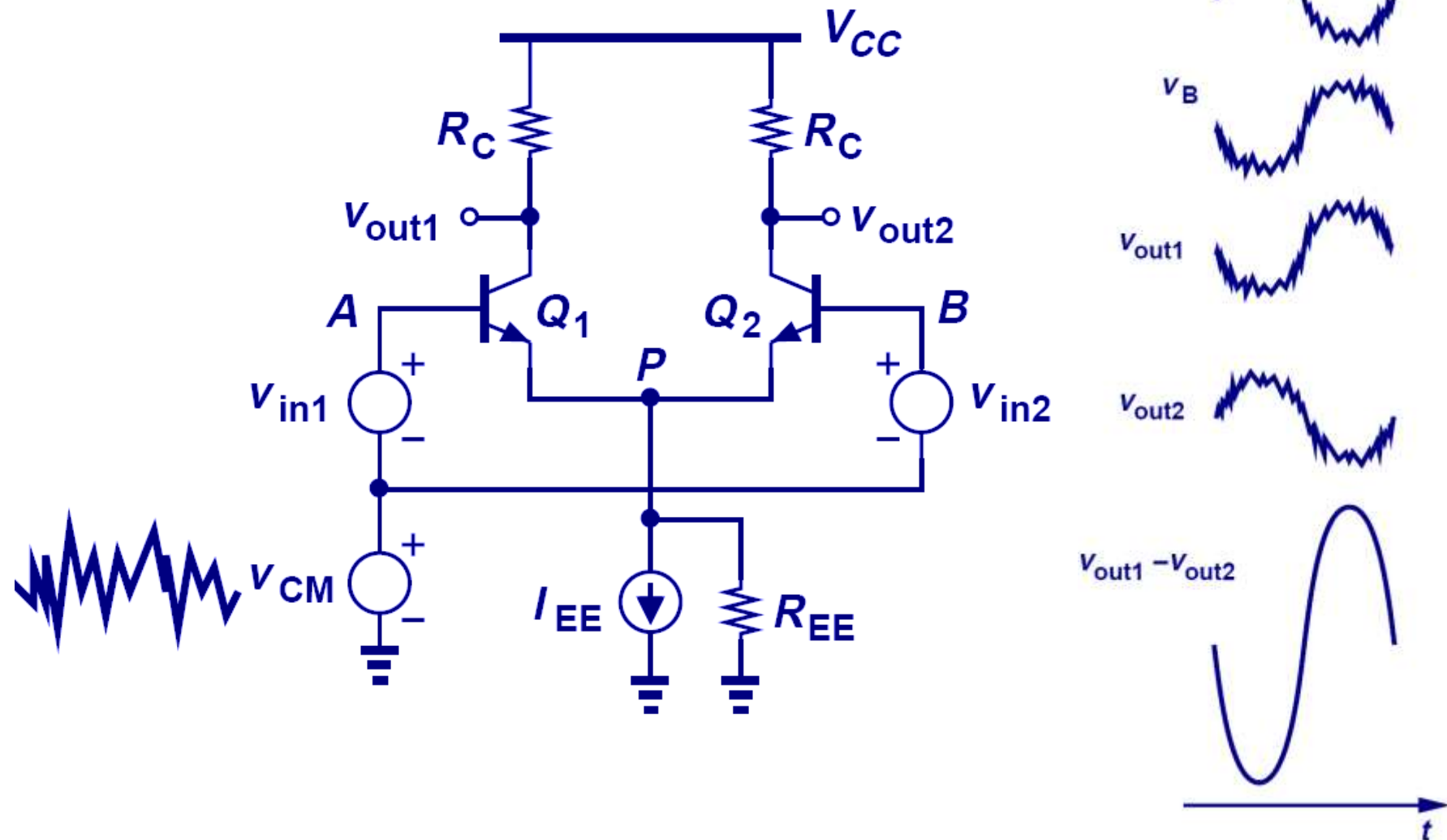
## Input CM Noise with Ideal Tail Current





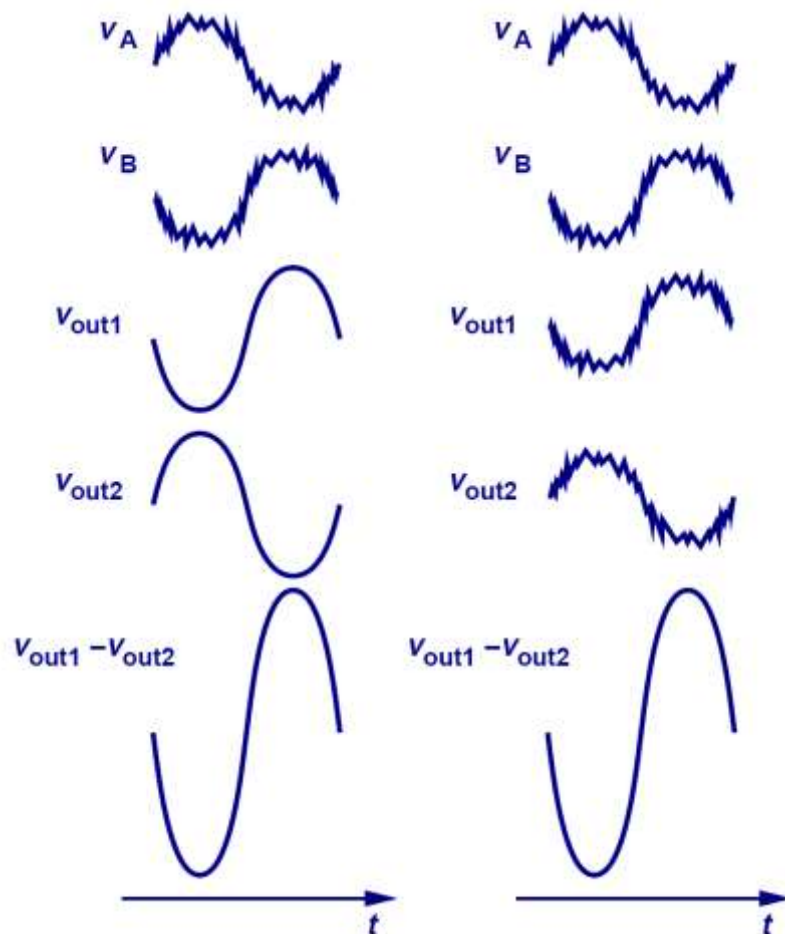
دانشگاه گیلان  
Guilan University

# Input CM Noise with Non-ideal Tail Current





## Comparison

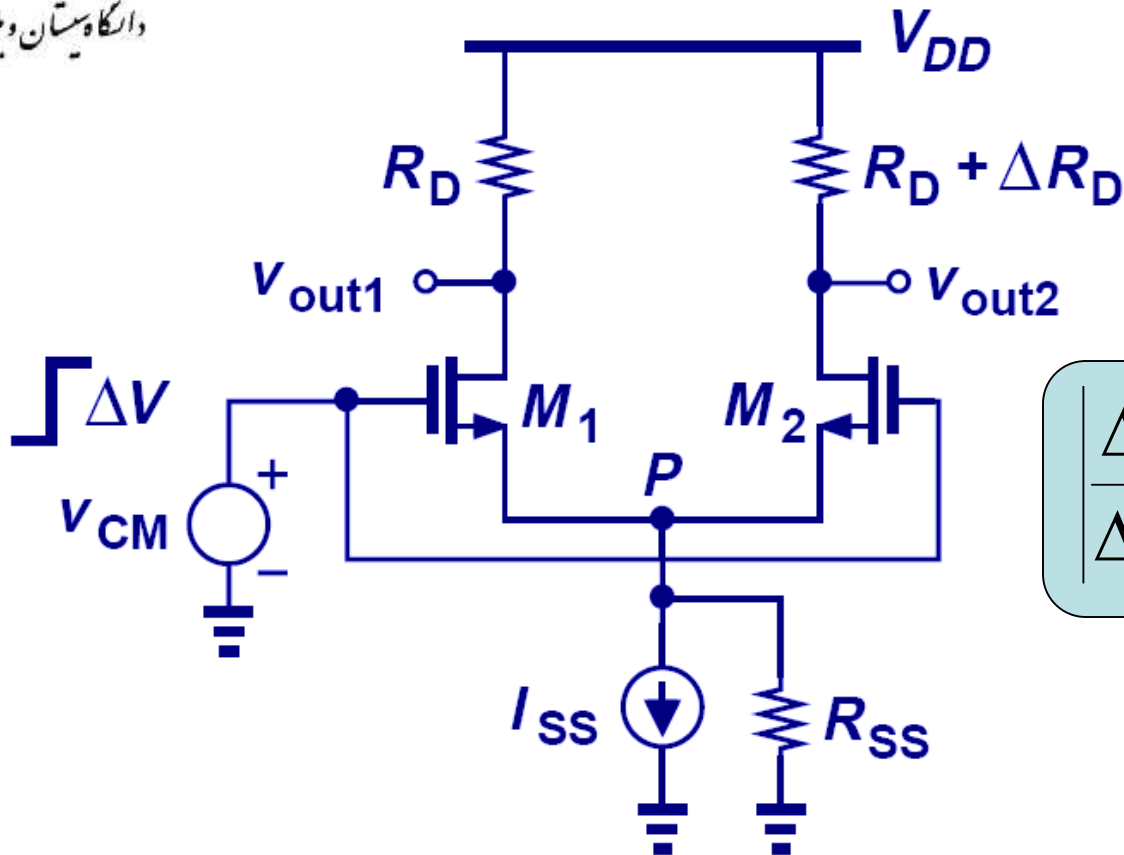


- As it can be seen, the differential output voltages for both cases are the same. So for small input CM noise, the differential pair is not affected.



دانشگاه پشاور

## CM to DM Conversion, $A_{CM-DM}$



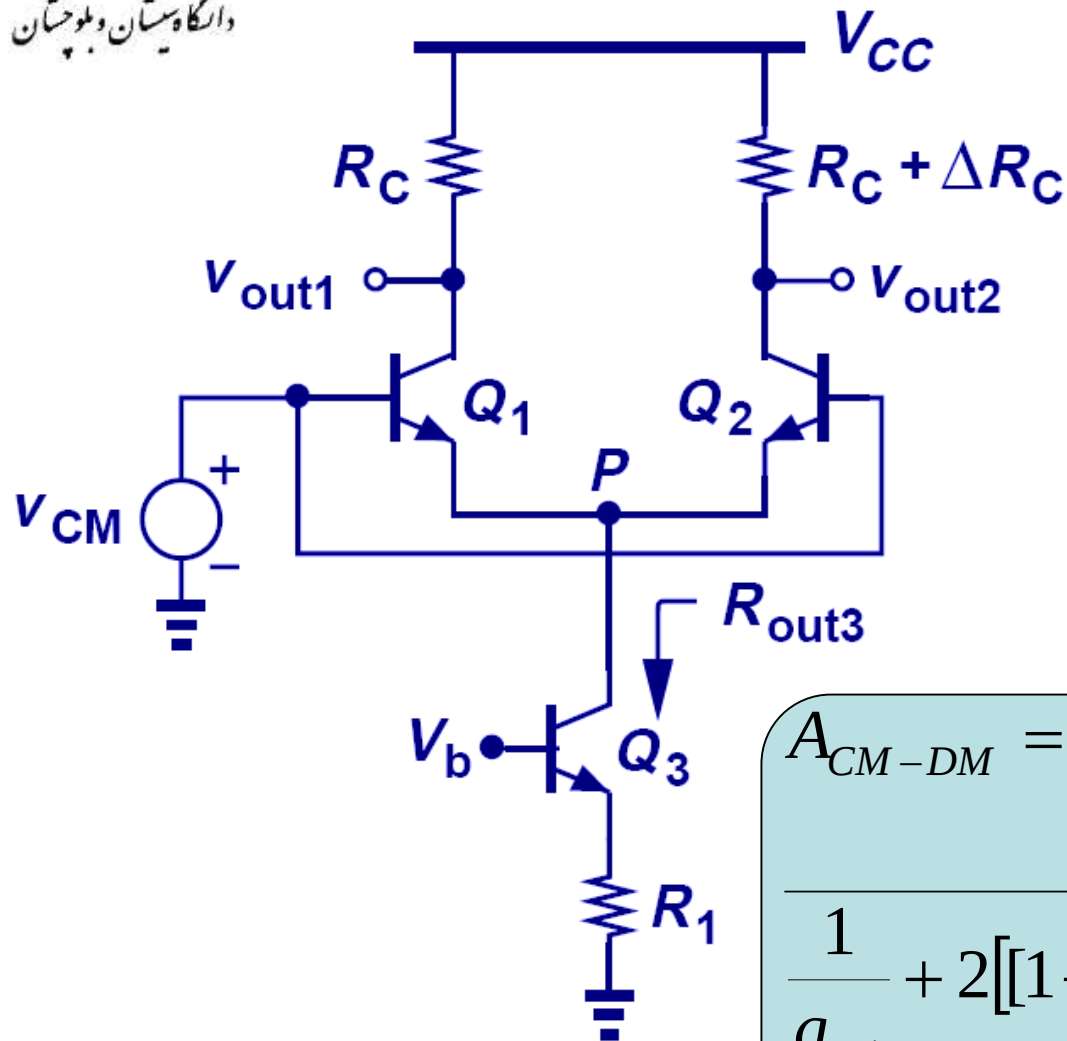
$$\left| \frac{\Delta V_{out}}{\Delta V_{CM}} \right| = \frac{\Delta R_D}{1/g_m + 2R_{EE}}$$

- If finite tail impedance and asymmetry are both present, then the differential output signal will contain a portion of input common-mode signal.



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## Example: $A_{CM-DM}$



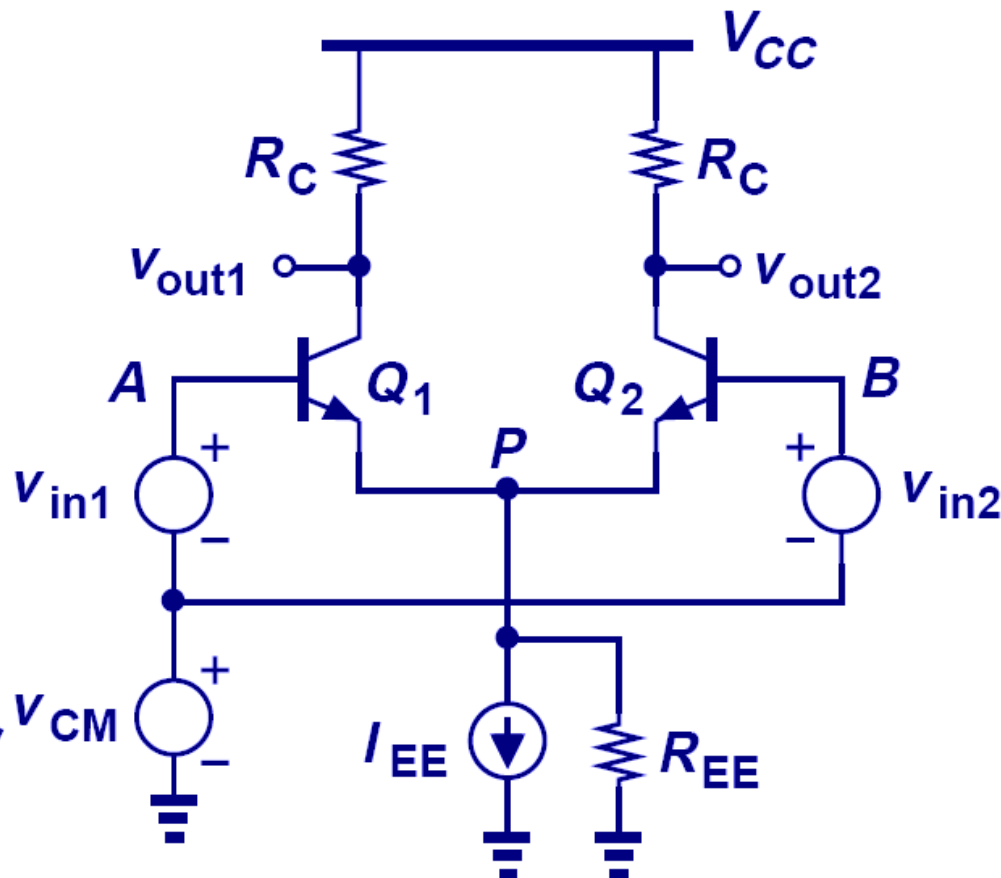
$$A_{CM-DM} =$$

$$\frac{\Delta R_C}{$$

$$\frac{1}{g_{m1}} + 2[1 + g_{m3}(R_1 \parallel r_{\pi 3})]r_{O3} + R_1 \parallel r_{\pi 3}]$$



## CMRR



$$CMRR = \frac{A_{DM}}{A_{CM-DM}}$$

- CMRR defines the ratio of wanted amplified differential input signal to unwanted converted input common-mode noise that appears at the output.