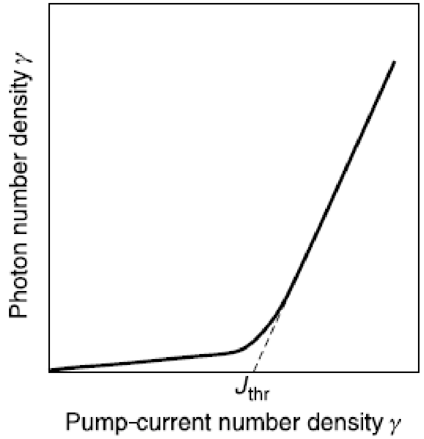
The Power–Current Curves

P–I or L–I curves.



Photon Density versus Pump-Current Number Density

$$\frac{dn}{dt} = -v_g\, g(n)\, \gamma + \mathscr{J} - Bn^2$$

$$\frac{d\gamma}{dt} = +\Gamma v_g\, g(n)\, \gamma - \frac{\gamma}{\tau_\gamma} + \beta B n^2$$

By: Dr. Jabbari

steady state

$$0 = -v_g g(n)\gamma + \mathscr{J} - Bn^2$$

$$0 = +\Gamma v_g g(n)\gamma - \frac{\gamma}{\tau_\gamma} + \beta Bn^2$$

Below Lasing Threshold

The phrase "below lasing threshold" implies that the laser has insufficient gain to support oscillation

$\mathscr{J} < \mathscr{J}_{\text{thr}}$

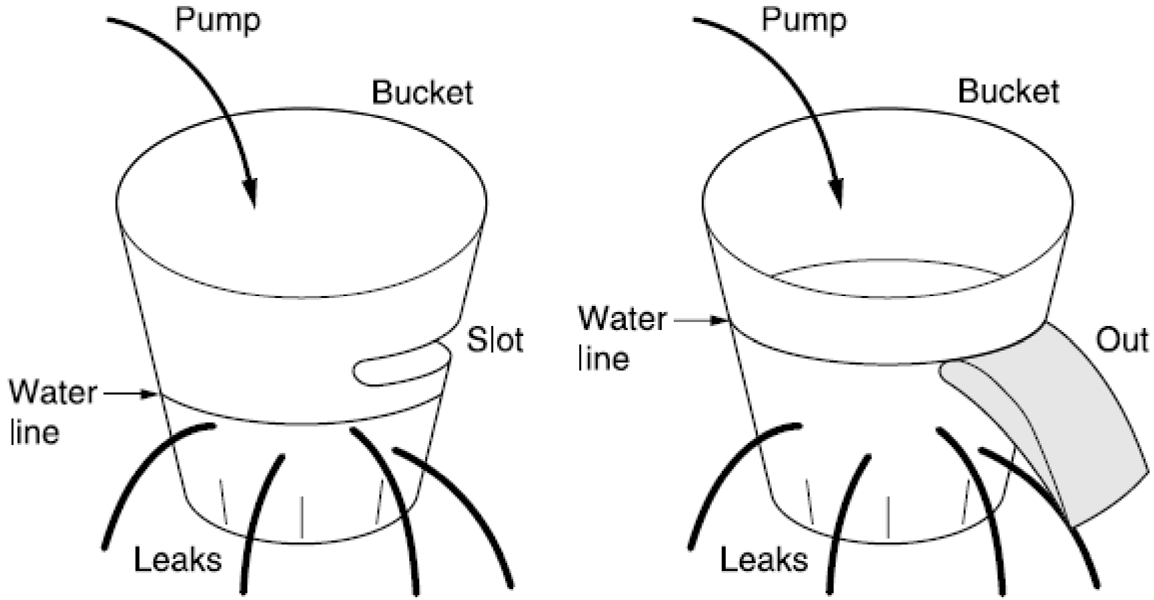
در حالت پایدار

g=0

$$\gamma = \beta\tau_\gamma Bn^2$$

$$Bn^2 = \mathscr{J}$$

$$\Rightarrow \gamma = \beta\tau_\gamma \mathscr{J}$$



$$\gamma = \beta \tau_\gamma B n^2$$

$$B n^2 = \mathscr{J}$$

$$\gamma = \beta \tau_\gamma \mathscr{J}$$

Above Lasing Threshold $(\bar{J} > \bar{J}_{\text{thr}})$

$$0 = -v_g g(n)\gamma + \mathscr{J} - R_{\text{rec}}$$

$$0 = +\Gamma v_g g(n)\gamma - \frac{\gamma}{\tau_\gamma}$$

$$R_{\text{rec}} = \frac{n}{\tau_e} = \frac{n}{\tau_n} + Bn^2 + Cn^3 \cong Bn^2$$

$$\frac{\text{Power out}}{\text{two mirrors}} = P_o = \frac{\text{Energy}}{\text{Sec}} = \left(\frac{\text{Energy}}{\text{Photon}} * \frac{\text{Photons}}{\text{Volume}} * \text{Mode volume}\right) * \frac{1}{\tau_m}$$

mirror time constant is

$$1/\tau_m = v_g \alpha_m.$$

energy of each photon

photons striking a single mirror must be proportional to γ/2 and for both mirrors

modal volume

$$P_o = \gamma \frac{hc}{\lambda_o} V_\gamma v_g \alpha_m$$

wavelength in vacuum

Case 1 P–I Below Threshold

We have:

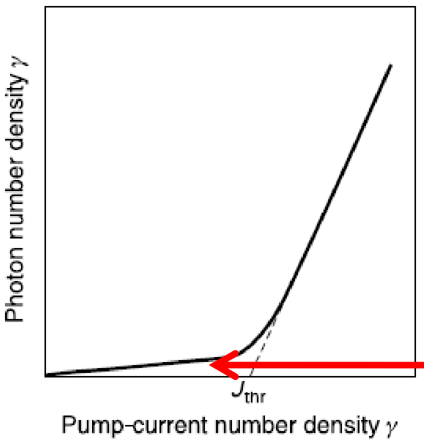
$$\gamma = \beta \tau_\gamma \mathscr{J}$$

$$P_o = \gamma \frac{hc}{\lambda_o} V_\gamma v_g \alpha_m$$

$$P_o = (\beta \tau_\gamma \mathscr{J}) \frac{hc}{\lambda_o} V_\gamma v_g \alpha_m$$

Next life time is:

$$\frac{1}{\tau_\gamma} = \frac{1}{\tau_m} + \frac{1}{\tau_{\text{int}}} = v_g(\alpha_m + \alpha_{\text{int}})$$



$$P_o = \beta \tau_\gamma \frac{\eta_i I}{qV} \frac{hc}{\lambda_o} V_\gamma v_g \alpha_m = \beta \frac{\eta_i}{q\Gamma} \frac{hc}{\lambda_o} \frac{\alpha_m}{\alpha_m + \alpha_{\text{int}}} I$$

Case 2 P–I Above Threshold

$$I > I_{\text{thr}}$$

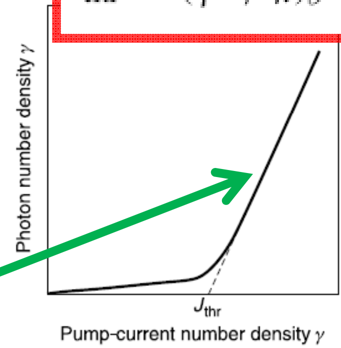
$$P_o = \gamma \frac{hc}{\lambda_o} V_\gamma v_g \alpha_m$$

$$J = \eta_i I/(qV),$$

$$\gamma = \Gamma\tau_\gamma(\mathscr{J} - \mathscr{J}_{\text{thr}})$$

$$\frac{P_o}{(hc/\lambda_o)\gamma V_\gamma v_g \alpha_m} = \Gamma\tau_\gamma\left(\frac{\eta_i I}{qV} - \frac{\eta_i I_{\text{thr}}}{qV}\right)$$

$$\boxed{I_{\text{thr}} = (qV/\eta_i)\mathscr{J}_{\text{thr}}}$$



با در نظر گرفتن موارد زیر:

$$\Gamma = V/V_\gamma$$

$$\frac{1}{\tau_\gamma} = \frac{1}{\tau_m} + \frac{1}{\tau_{\text{int}}} = v_g(\alpha_m + \alpha_{\text{int}})$$

$$P_o = \eta_i \frac{hc}{q\lambda_o} \frac{\alpha_m}{\alpha_{\text{int}} + \alpha_m}(I - I_{\text{thr}})$$

Power versus Voltage

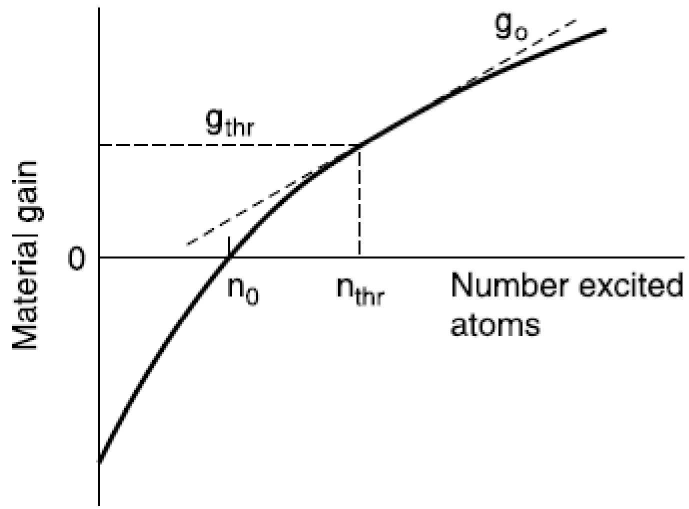
$$I = I_o\left(e^{qV/kT} - 1\right) \sim I_o\, e^{qV/kT}$$

Some Comments on Gain

$$g_{\text{thr}} = g(n_{\text{thr}}) = g_o(n_{\text{thr}} - n_o).$$

$$g_o(n) = \frac{dg(n)}{dn}$$

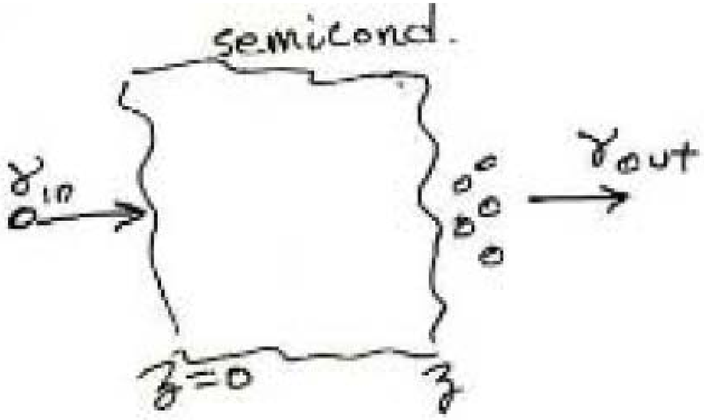
$$g(n) = g_o(n - n_o)$$



By: Dr. Jabbari

Material trancparency

بدون در نظر گرفتن انعکاس و پراکندگی و بدون در نظر گرفتن آینه ها



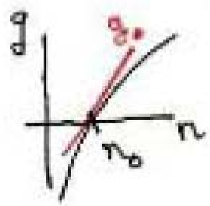
$$\frac{d\gamma}{dt} \simeq v_g\, g\, \gamma \Rightarrow \frac{d\gamma}{dz} = g\gamma$$

$$\Rightarrow \gamma_{out} = \gamma_{in}\, e^{gz}$$

Trancparency occur when $\gamma_{out} = \gamma_{in}$

$$1 = \frac{\gamma_{out}}{\gamma_{in}} = \underbrace{G}_{\text{single pass}} = e^{gz} \rightarrow g \simeq 0$$

$$n = n_0 = \text{transp. carr. density}$$

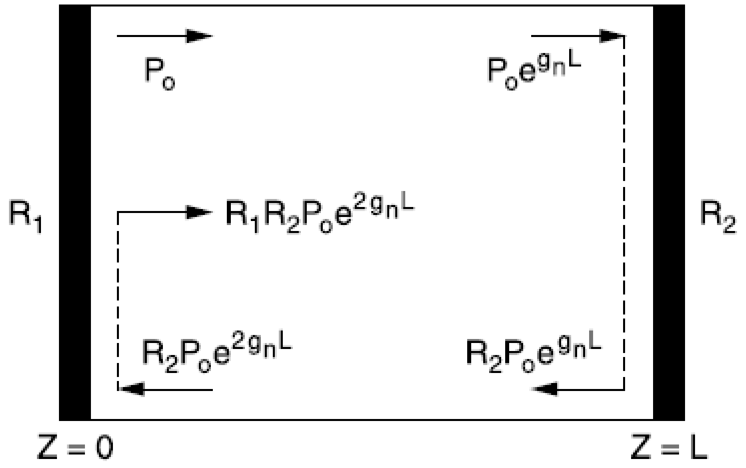


$$g(n) \simeq g_0 (n - n_0)$$

$$g_0 = \frac{dg}{dn} = \text{different. gain}$$

Relations for Cavity Lifetime, Reflectances and Internal Loss

mirror *mirror*



R=power Reflectances

Lock at round trip

Relation between P1 and P2 with only material gain and internal loss

$$\frac{d\gamma}{dt} = +\Gamma v_g g(n)\gamma - \frac{\gamma}{\tau_\gamma}$$

$$\frac{1}{\tau_\gamma} = \underbrace{v_g \alpha_{\text{int}}}_{\text{internal losses}} + v_g \alpha_m = v_g \alpha_{\text{int}} + \frac{v_g}{2L}\text{Ln}\left(\frac{1}{R_1 R_2}\right)$$

$$\frac{1}{\tau_\gamma} = \frac{1}{\tau_{\text{int}}} = v_g \alpha_{\text{int}}$$

the cavity lifetime

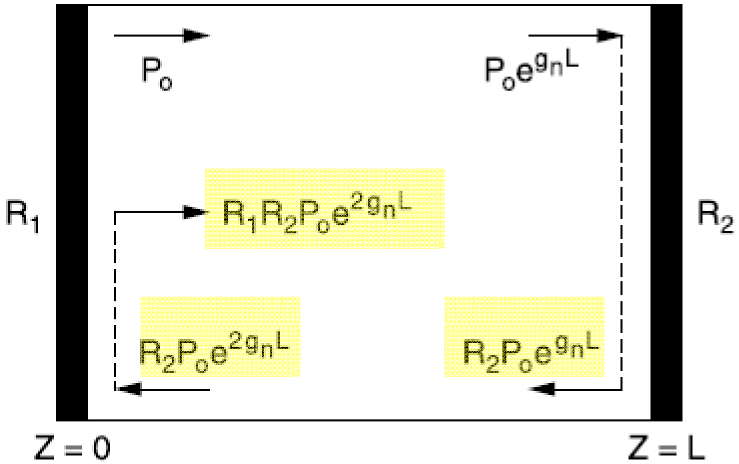
No reflection facet $a_{m=0}$

$$\frac{d\gamma}{dt} = v_g \frac{d\gamma}{dz} = v_g g\gamma - \gamma v_g \alpha_{\text{int}} \quad \text{or} \quad \frac{d\gamma}{dz} = g\gamma - \gamma\alpha_{\text{int}} = g_{\text{net}}\gamma$$

where the net gain $g_{\text{net}} = g - \alpha_{\text{int}}$

So we have $\frac{dP}{dz} = g_{\text{net}} P$ $\Rightarrow$ $P(z) = P_o \exp(g_n z)$

from $z = 0$ to $z = L$.

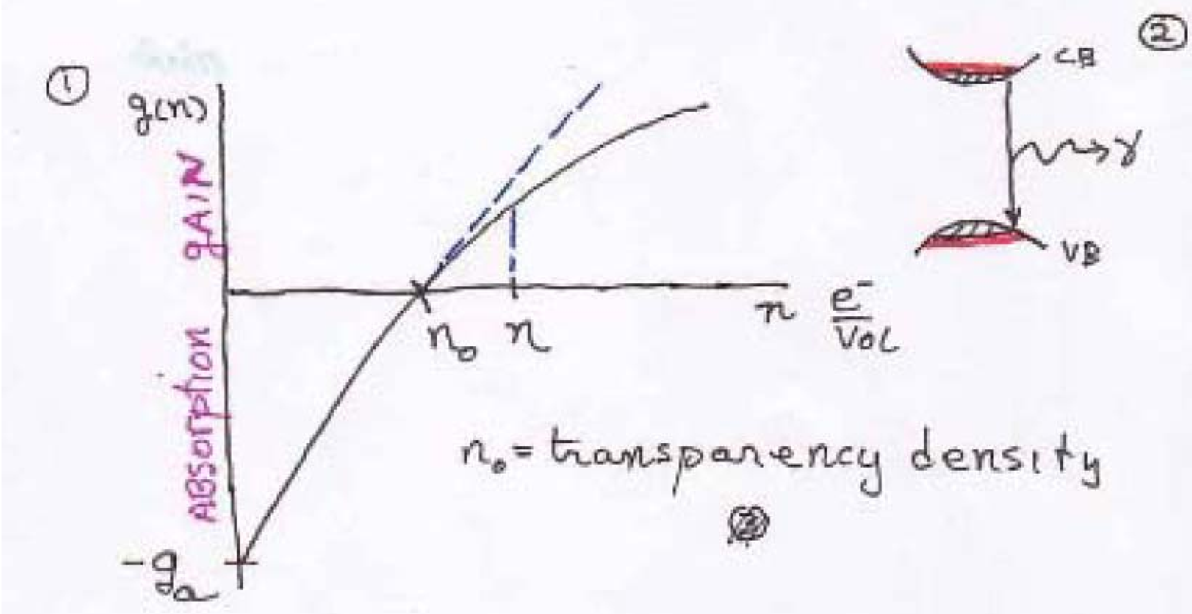


For steady state the initial power Po must be the same as the final power

$$P_o = R_1 R_2 P_o \exp(2 g_n L)$$

$$g_{\text{net}} = \frac{1}{2L} \text{Ln}\left(\frac{1}{R_1 R_2}\right)$$

Some BASIC FACTS About GaiN



- Recall $\gamma \sim e^{gL}\gamma_0$ g = material only

  $g = 0 \rightarrow \gamma = \gamma_0$ NOT $g_{net} = g - \alpha_{int}$

  Transparency occurs at n_0

  1 photon 'in' $\rightarrow$ 1 photon 'out'

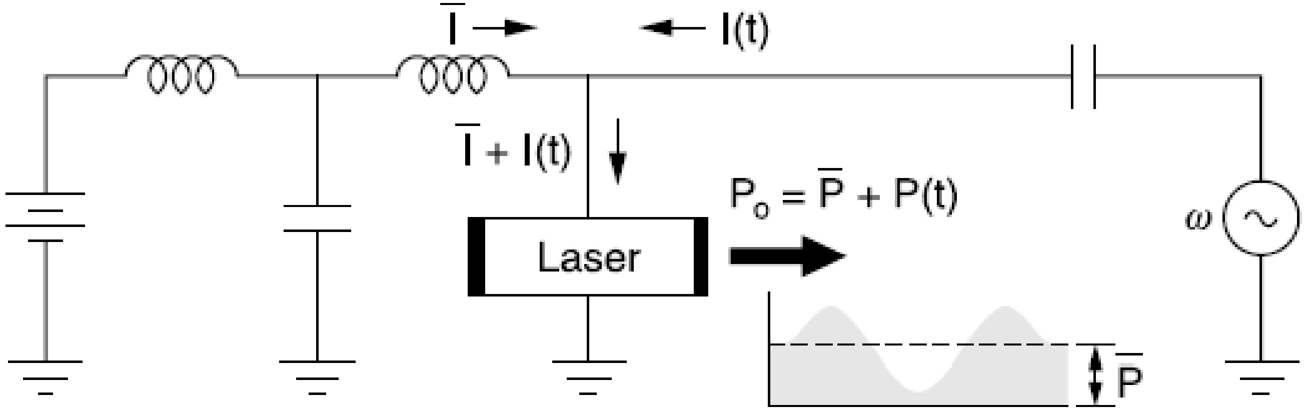
- Can linearize $g = g(n)$ by Taylor Expanding About n_0

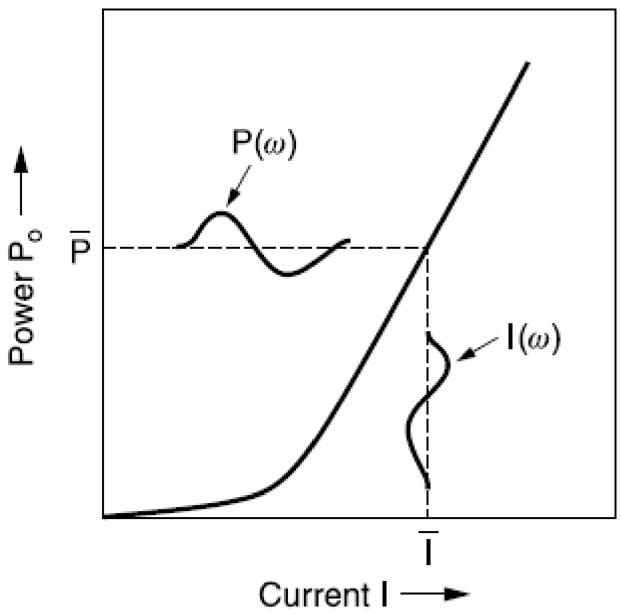
$$\boxed{g \sim g_0'(n - n_0)}$$

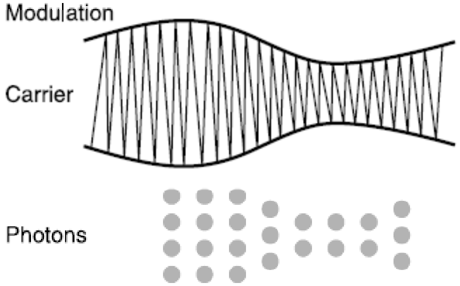
- $n < n_0 \rightarrow g < 0 \longrightarrow$ $\gamma = e^{gL}\gamma_0 < \gamma_0$ Absorp
- $n > n_0 \rightarrow g > 0 \longrightarrow$ $\gamma = e^{gL}\gamma_0 > \gamma_0$ Gain
- $n = n_0 \rightarrow g = 0 \longrightarrow$ $\gamma = \gamma_0$ Transparency

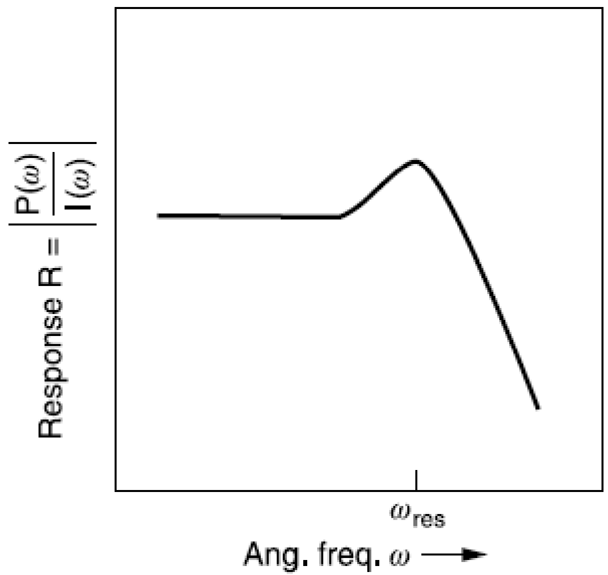
Modulation Bandwidth

$$I(t) = I\,e^{i\omega t}$$









$$\omega_{res} = \sqrt{\frac{v_g g_o \bar{\gamma}}{\tau_\gamma}}$$

By: Dr. Jabbari

Small Signal Analysis

To demonstrate the equation for resonance

“bar’ ‘’average’

Rate and Wave Equation

$$\frac{dn'}{dt} = -v_g g_o(n' - n_o)\gamma' + J' - \frac{n'}{\tau_n}$$

$$\frac{d\gamma'}{dt} = +\Gamma v_g g_o(n' - n_o)\gamma' - \frac{\gamma'}{\tau_\gamma}$$

$$n' = \bar{n} + n\, e^{i\omega t} \qquad \gamma' = \bar{\gamma} + \gamma\, e^{i\omega t} \qquad J' = \bar{J} + J\, e^{i\omega t}$$

For steady-state

$$\begin{cases} 0 = -v_g g_o(\bar{n} - n_o)\bar{\gamma} + \bar{J} - \dfrac{\bar{n}}{\tau_n} \\ 0 = +\Gamma v_g\, g_o(\bar{n} - n_o)\,\bar{\gamma} - \dfrac{\bar{\gamma}}{\tau_\gamma} \end{cases}$$

$$\begin{cases} i\omega n\, e^{i\omega t} = -v_g\left(\bar{g} + g_o n\, e^{i\omega t}\right)\left(\bar{\gamma} + \gamma\, e^{i\omega t}\right) + \bar{J} - \dfrac{\bar{n}}{\tau_n} + J\, e^{i\omega t} - \dfrac{n}{\tau_n} e^{i\omega t} \\ i\omega\gamma\, e^{i\omega t} = \Gamma v_g\left(\bar{g} + g_o n\, e^{i\omega t}\right)\left(\bar{\gamma} + \gamma\, e^{i\omega t}\right) - \dfrac{\gamma\, e^{i\omega t}}{\tau_\gamma} - \dfrac{\bar{\gamma}}{\tau_\gamma} \end{cases}$$

$$\begin{cases} i\omega n\, e^{i\omega t} = -v_g\left[g_o n\, e^{i\omega t}\, \bar{\gamma} + \bar{g}\gamma\, e^{i\omega t} + g_o\gamma n\, e^{i\omega t}\right] + J\, e^{i\omega t} - \dfrac{n}{\tau_n} e^{i\omega t} \\ i\omega\gamma\, e^{i\omega t} = \Gamma v_g\left[\bar{\gamma} g_o n\, e^{i\omega t} + \bar{g}\gamma\, e^{i\omega t} + g_o n\gamma\, e^{2i\omega t}\right] - \dfrac{\gamma\, e^{i\omega t}}{\tau_\gamma} \end{cases}$$

Drop the second-order nonlinear terms such as g_n, n^2; this procedure also removes termssuch as e(2iw). Then cancel the exponentials e(iw) from both sides

$$\begin{cases} i\omega n = -v_g g_o n\bar{\gamma} - v_g \bar{g}\gamma + J - \dfrac{n}{\tau_n} \\ i\omega\gamma = \Gamma v_g g_o n\bar{\gamma} + \Gamma v_g \bar{g}\gamma - \dfrac{\gamma}{\tau_\gamma} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} v_g \bar{g}\gamma = J - \left(v_g g_o \bar{\gamma} + \dfrac{1}{\tau_n} + i\omega \right) n \\ \Gamma v_g g_o \bar{\gamma} n = \left(\dfrac{1}{\tau_\gamma} - \Gamma v_g \bar{g} + i\omega \right) \gamma \end{cases}$$

$$\gamma = \frac{\Gamma v_g g_o \bar{\gamma}\, J}{\Gamma v_g^2 g_o \bar{g}\bar{\gamma} + \left(v_g g_o \bar{\gamma} + \frac{1}{\tau_n} + i\omega \right)\left(\frac{1}{\tau_\gamma} - \Gamma v_g \bar{g} + i\omega \right)}$$

define

$$\begin{cases} \frac{1}{\tau} = \left(\frac{1}{\tau_\gamma} - \Gamma v_g \bar{g} + v_g g_o \bar{\gamma} + \frac{1}{\tau_n} \right) \\ \omega_o^2 = \Gamma v_g^2 g_o \bar{g}\bar{\gamma} + \left(v_g g_o \bar{\gamma} + \frac{1}{\tau_n} \right)\left(\frac{1}{\tau_\gamma} - \Gamma v_g \bar{g} \right) \end{cases}$$

obtain the transfer function

$$\frac{\gamma}{J} = \frac{\Gamma v_g g_o \bar{\gamma}}{\left(\omega_o^2 - \omega^2 \right) + \left(i\omega / \tau \right)}$$

Defined the response function

$(\Gamma v_g g_o \bar{\gamma})^2$

$R_J =$

peak of
respons
functio

- Better mirrors lowers B.W.
- Larger differential Gain (a material property) increase BW
- Longer Lasers are Slower

Really a trade off according to $\frac{\ln R}{L}$

Above t

$$\frac{1}{\tau_\gamma} = \Gamma v_g \bar{g} \quad \rightarrow \quad \Gamma v_g \bar{g} \bar{\gamma} = \frac{\bar{\gamma}}{\tau_\gamma}$$

$$\omega_{res} = \sqrt{g_o v_g^2 \bar{\gamma} \left[\alpha_{int} + \frac{1}{L} \mathrm{Ln}\left(\frac{1}{R}\right) \right]}$$

- Better Mirrors Lowers B.W.
- Larger differential Gain (a material property) increase BW
- Longer Lasers are Slower

  Really a trade off according to $\frac{\ln R}{L}$